ELSEVIER

1 October 1997

Optics Communications 142 (1997) 91–98

OPTICS COMMUNICATIONS

Full length article

Comparison between various formulations of the extended boundary condition method

T. Wriedt [a], A. Doicu [b]

[a] *Institut für Werkstofftechnik, Badgasteiner Strasse 3, 28359 Bremen, Germany*
[b] *Fachgebiet Verfahrenstechnik, Universität Bremen, Badgasteiner Strasse 3, 28359 Bremen, Germany*

Received 1 April 1997; revised 23 May 1997; accepted 23 May 1997

Abstract

The accuracy of various formulations of the extended boundary condition method (EBCM) for solving the scattering problem of a cone-sphere particle is investigated. We compare the standard EBCM, the multiple multipole EBCM, the lowest-order multipole EBCM and the EBCM with sub-domain bases. For this purpose we choose the residual of the total electric field on spherical surfaces with shifted origins as error criterion. A hybrid method consisting of a combination of sub-boundary bases and global-boundary bases for the surface current density representation is also discussed. © 1997 Elsevier Science B.V.

1. Introduction

To describe the process of electromagnetic wave propagation in the presence of local scatterers, it is necessary to solve the boundary value problems of diffraction theory. An effective technique for analyzing the scattered field from an object was proposed by Waterman [1,2]. This approach is known as the extended boundary condition method (EBCM) since a homogeneous condition of the unknown functions in the null-field region is used to formulate the integral equation. The substance of this method is an eigen-based boundary integral equation method weighted by Green's functions of unbounded space.

Even though the application of the standard EBCM is widely considered to be quite successful for treating problems that involve the far-field scattering characteristics a major limiting factor lies in the use of EBCM for modeling the internal fields, especially inside highly elongated particles. As shown by Bates [3] it is usually easier to compute far fields, rather than surface fields to a specified accuracy. This is because surface fields require an accurate description to satisfy the boundary conditions, which has negligible effect in the far field. Consequently, far fields computed by the standard EBCM are often satisfactory when the corresponding computations of near fields and surface sources are inadequate. However, numerical instabilities of the scattered field computed with the standard, single spherical coordinate-based EBCM appear for scattering problems involving particles with large aspect ratios, or particles whose surfaces have appreciable concavities, or geometrical singularities. A number of modifications to this method have been suggested to ameliorate these problems, including improved numerical methods [2,4,5], formal modifications of the single spherical coordinate-based EBCM [6–8], different choices of basis functions [9], and the application of the spheroidal coordinate formalism [10].

The aim of this paper is to investigate the feasibility of using different formulations of the EBCM for solving the scattering problem from a cone-sphere particle. More precisely we analyze the applicability of the standard EBCM, the multiple multipole EBCM [11], the lowest-order multipole EBCM [12] and the EBCM with sub-domain bases [9,13,14]. We note here that, in the case of tips or edges on the main surface, the fields should have singularities near the points of geometrical singularities. The usual technique is to smooth the geometrical singularities with certain radii

PII S0030-4018(97)00278-2

of curvature, or to incorporate surface current densities capable of representing the correct singularity in subdomains of interest. In the EBCM a simple technique which consists of computing the surface integrals using the Gaussian quadrature is often used [15,16]. For particles consisting of more than one section one defines quadrature sample points and weighting values over each section separately, since the Gaussian quadrature will not select the end points of integration. Formally, this technique is equivalent to the procedure of smoothing geometrical singularities.

2. Formulation

Let us consider the scattering problem of an incident field ($\bar{E}_0$, $\bar{H}_0$) by a homogeneous dielectric object with surface S. The interior of S is denoted by Ω_i, and its exterior by Ω_s. We choose a point O within Ω_i to be the origin of a Cartesian coordinate system $Oxyz$. An arbitrary point in Ω is denoted by the position vector $\bar{r}$, while an arbitrary point on S is given by $\bar{r}'$. We denote by k_t the wave number of the region Ω_t, where, $k_t = k\sqrt{\varepsilon_t \mu_t}$, $t = s,i$ and $k = \omega/c$.

In the EBCM the scattering problem is divided into an external and an internal subproblem for the surface current densities. For each subproblem, surface current densities are generated to create the actual field in the region of interest and a null-field elsewhere. Since the fields are zero outside the region of interest, it makes no difference how the dielectric characteristics in these regions are and therefore they can be assigned with the same characteristics as the region where the actual fields exist. For the external subproblem the scattering object is replaced by a set of surface current densities $\bar{n} \times \bar{E}_+$ and $\bar{n} \times \bar{H}_+$ on the surface S, so that in the exterior region the sources and fields are exactly the same as those existing in the original scattering problem. The expression for the total field is

$$\begin{pmatrix} \bar{E}(\bar{r}) \\ 0 \end{pmatrix} = \bar{E}_0(r) + \nabla \times \int_S \left(\bar{n} \times \bar{E}_+(\bar{r}')\right) g\left(k_S|\bar{r} - \bar{r}'|\right) \mathrm{d}S' - \nabla \times \nabla \times \int_S \frac{1}{jk\varepsilon_S} \left(\bar{n} \times \bar{H}_+(\bar{r}')\right) g\left(k_S|\bar{r} - \bar{r}'|\right) \mathrm{d}S' \quad \text{for} \begin{pmatrix} \bar{r} \in \Omega_S \\ \bar{r} \in \Omega_i \end{pmatrix}, \tag{1}$$

where $g(k_S|\bar{r} - \bar{r}'|)$ is the free space Green's function and $\bar{n}$ is the outward unit normal to S. For the internal subproblem, the surface current densities $\bar{n} \times \bar{E}_-$ and $\bar{n} \times \bar{H}_-$ are set up to create the internal field inside the scatterer and a null-field outside, i.e.

$$\begin{pmatrix} \bar{E}_i(\bar{r}) \\ 0 \end{pmatrix} = \nabla \times \int_S \left(-\bar{n} \times \bar{E}_-(\bar{r}')\right) g\left(k_i|\bar{r} - \bar{r}'|\right) \mathrm{d}S' - \nabla \times \nabla \times \int_S \frac{1}{jk\varepsilon_i} \left(-\bar{n} \times \bar{H}_-(\bar{r}')\right) g\left(k_i|\bar{r} - \bar{r}'|\right) \mathrm{d}S' \quad \text{for} \begin{pmatrix} \bar{r} \in \Omega_i \\ \bar{r} \in \Omega_S \end{pmatrix}. \tag{2}$$

Relationships between the subproblems can be obtained by considering the continuity of the tangential components of the electric and magnetic fields across the surface S, i.e.

$$\bar{n} \times \bar{E}_+ = \bar{n} \times \bar{E}_- = \bar{e}, \quad \bar{n} \times \bar{H}_+ = \bar{n} \times \bar{H}_- = \bar{h}. \tag{3}$$

In order to simplify our presentation we focus our analysis on scatterers with axial symmetry and choose the z-axis to be the symmetry axis of the particle. Let us consider a rigid translation z_p of the original coordinate system along the z axis and denote the origin of the new coordinate system by O_p, so that $\bar{r}_p = \bar{r} - z_p \bar{e}_3$. Here, $\bar{e}_i$ is the Cartesian system basis. The total electric field can be expressed in the translated coordinate system O_p as a series of spherical vector wave functions (SVWF) of the first kind $\bar{M}^1_{mn}(k_S \bar{r}_p)$ and $\bar{N}^1_{mn}(k_S \bar{r}_p)$, i.e.

$$\bar{E}(\bar{r}_p + z_p \bar{e}_3) = \sum_{m \in Z} \sum_{n \geq \max(1,|m|)} D_{mn} \left[a^{(p)}_{mn}(\bar{e} - \bar{e}_0, \bar{h} - \bar{h}_0) \times \bar{M}^1_{mn}(k_S \bar{r}_p) + b^{(p)}_{mn}(\bar{e} - \bar{e}_0, \bar{h} - \bar{h}_0) \bar{N}^1_{mn}(k_S \bar{r}_p) \right], \tag{4}$$

where D_{mn} is a normalization constant and $\bar{e}_0 = \bar{n} \times \bar{E}_0$ and $\bar{h}_0 = \bar{n} \times \bar{H}_0$. The representation given in Eq. (4) is valid inside an inscribed sphere with center O_p. The expansion coefficients $a^{(p)}_{mn}(\bar{e},\bar{h})$ and $b^{(p)}_{mn}(\bar{e},\bar{h})$ are expressed as surface integrals and are given by

$$a^{(p)}_{mn}(\bar{e},\bar{h}) = \frac{jk_S^2}{\pi} \int_S \left[\bar{e}(\bar{r}') \cdot \bar{N}^{(3)}_{-m,n}(k_S \bar{r}'_p) + j\sqrt{\frac{\mu_S}{\epsilon_S}}\, \bar{h}(\bar{r}') \cdot \bar{M}^{(3)}_{-m,n}(k_S \bar{r}'_p) \right] \mathrm{d}S',$$

$$b^{(p)}_{mn}(\bar{e},\bar{h}) = \frac{jk_S^2}{\pi} \int_S \left[\bar{e}(\bar{r}') \cdot \bar{M}^{(3)}_{-m,n}(k_S \bar{r}'_p) + j\sqrt{\frac{\mu_S}{\epsilon_S}}\, \bar{h}(\bar{r}') \cdot \bar{N}^{(3)}_{-m,n}(k_S \bar{r}'_p) \right] \mathrm{d}S'. \tag{5}$$

By convention, $a^{(0)}_{mn}(\bar{e} - \bar{e}_0, \bar{h} - \bar{h}_0)$ and $b^{(0)}_{mn}(\bar{e} - \bar{e}_0, \bar{h} - \bar{h}_0)$ represent the expansion coefficients with respect to the original coordinate system $Oxyz$. Analogously, we denote by $c_{mn}^{(p)}(\bar{e},\bar{h})$ and $d_{mn}^{(p)}(\bar{e},\bar{h})$ the expansion coefficients of the internal field in terms of SVWF of the third kind $\overline{M}_{mn}^{3}(k_i\bar{r}_p)$ and $\overline{N}_{mn}^{3}(k_i\bar{r}_p)$. Again, this representation is valid outside a circumscribed sphere with center O_p.

Various formulations of the EBCM are summarized below.

(i) *Standard EBCM*

A quasi-solution of the scattering problem can be obtained by approximating the surface current densities in the mean square norm by the complete set of tangential, single spherical coordinate vector wave functions of the internal problem and by considering the null-field condition for the total electric field within Ω_i. In fact the amplitudes of an approximate solution of the surface current densities

$$\begin{pmatrix} \bar{e}^{\hat{N}}(\bar{r}') \\ \bar{h}^{\hat{N}}(\bar{r}') \end{pmatrix} = \sum_{m=-M}^{M} \sum_{n>\max(1,|m|)}^{N} \alpha_{mn} \begin{pmatrix} \bar{n}\times \overline{M}_{mn}^{1}(k_i\bar{r}') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{N}_{mn}^{1}(k_i\bar{r}')) \end{pmatrix} + \beta_{mn} \begin{pmatrix} \bar{n}\times \overline{N}_{mn}^{1}(k_i\bar{r}') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{M}_{mn}^{1}(k_i\bar{r}')) \end{pmatrix} \tag{6}$$

can be obtained as a solution of the truncated system of integral equations

$$a_{mn}^{(0)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0,$$
$$b_{mn}^{(0)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0,\quad m=\overline{-M,M},$$
$$n=\overline{\max(1,|m|),N}. \tag{7}$$

Here, $\hat{N}$ is a complex index incorporating M and N.

(ii) *Multiple multipole EBCM*

Let us consider a finite collection of poles $\{z_p\}_{p=\overline{1,N_p}}$. Then, the amplitudes of the surface current densities written in an explicit form as

$$\begin{pmatrix} \bar{e}^{\hat{N}}(\bar{r}') \\ \bar{h}^{\hat{N}}(\bar{r}') \end{pmatrix} = \sum_{p=1}^{N_p} \sum_{m=-M(p)}^{M(p)} \times \sum_{n\geq \max(1,|m|)}^{N(p)} \alpha_{mn}^{(p)} \begin{pmatrix} \bar{n}\times \overline{M}_{mn}^{1}(k_i\bar{r}_p') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{N}_{mn}^{1}(k_i\bar{r}_p')) \end{pmatrix} + \beta_{mn}^{(p)} \begin{pmatrix} \bar{n}\times \overline{N}_{mn}^{1}(k_i\bar{r}_p') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{M}_{mn}^{1}(k_i\bar{r}_p')) \end{pmatrix} \tag{8}$$

satisfy the truncated system of integral equations

$$a_{mn}^{(p)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0,$$
$$b_{mn}^{(p)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0,\quad p=1,\dots,N_p,$$
$$m=-M(p),\dots,M(p),$$
$$n=\max(1,|m|),\dots,N(p). \tag{9}$$

This formulation is similar to the Generalized Multipole Technique [17,18]. The surface current densities are approximated in the mean-square norm by normal multipole expansions with different origins, and the null-field condition for the exterior field is simultaneously imposed in different inscribed spheres. The main difference consists in the fact that this method is a combination of the duality of entire-domain bases for exterior fields and multiple-domain bases for interior fields. We note here that the locations of the spheres where the null-field condition is imposed may be different from the positions of the poles where the internal field approximation is considered. Furthermore, the position of the poles may not be restricted to the z-axis.

(iii) *Lowest-order multipole EBCM*

Let the set $\{z_p\}_{p=\overline{1,\infty}}=\Gamma_z$, where $\Gamma_z\subset\Omega_i$ is a segment of the z-axis. Under these circumstances the following scheme for surface current determination can be considered. The amplitudes of a quasisolution of the surface current densities

$$\begin{pmatrix} \bar{e}^{\hat{N}}(\bar{r}') \\ \bar{h}^{\hat{N}}(\bar{r}') \end{pmatrix} = \sum_{m=-M}^{M} \sum_{p=1}^{N} \alpha_{mp} \begin{pmatrix} \bar{n}\times \overline{M}_{m,|m|+l}^{1}(k_i\bar{r}_p') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{N}_{m,|m|+l}^{1}(k_i\bar{r}_p')) \end{pmatrix} + \beta_{mp} \begin{pmatrix} \bar{n}\times \overline{N}_{m,|m|+l}^{1}(k_i\bar{r}_p') \\ -j\sqrt{\varepsilon_i/\mu_i}\,(\bar{n}\times \overline{M}_{m,|m|+l}^{1}(k_i\bar{r}_p')) \end{pmatrix} \tag{10}$$

can be obtained as a solution of the truncated system of integral equations

$$a_{m,|m|+l}^{(p)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0$$
$$b_{m,|m|+l}^{(p)}\left(\bar{e}^{\hat{N}}-\bar{e}_0,\bar{h}^{\hat{N}}-\bar{h}_0\right)=0,$$
$$m=\overline{-M,M},\quad p=\overline{1,N}. \tag{11}$$

Here, l is a fixed index, given by $l=1$ if $m=0$ and $l=0$ otherwise. This formulation represents a modified version of the EBCM in which the surface current densities are approximated, for a fixed value of the azimuthal mode m, by the lowest-order multipoles located on the z axis. Again, the positions of the poles in Eqs. (10) and (11) may be different.

(iv) *EBCM with sub-domain bases*

A general formulation of the EBCM can be constructed if one considers the coupled null-field integral equations (1) and (2), and the surface current densities as independent unknowns. Under these circumstances, a trial solution of the surface current densities

$$\bar{e}^{\hat{N}}(\bar{r}') = \sum_{m=-M}^{M} \sum_{n=1}^{N'(m)} [\alpha_{mn}\Psi_n(\bar{r}'(\theta))\bar{\tau}_1 + \beta_{mn}\Psi_n(\bar{r}'(\theta))\bar{\tau}_2]\,\mathrm{e}^{jm\varphi},$$

$$\bar{h}^{\hat{N}}(\bar{r}') = -j\sqrt{\frac{\epsilon_i}{\mu_i}} \sum_{m=-M}^{M} \sum_{n=1}^{N'(m)} [\delta_{mn}\Psi_n(\bar{r}'(\theta))\bar{\tau}_1 + \gamma_{mn}\Psi_n(\bar{r}'(\theta))\bar{\tau}_2]\,\mathrm{e}^{jm\varphi}, \quad (12)$$

can be obtained by solving the external and the internal null-field equations, i.e.

$$a_{mn}^{(0)}\left(\bar{e}^{\hat{N}} - \bar{e}_0, \bar{h}^{\hat{N}} - \bar{h}_0\right) = 0,$$

$$b_{mn}^{(0)}\left(\bar{e}^{\hat{N}} - \bar{e}_0, \bar{h}^{\hat{N}} - \bar{h}_0\right) = 0,$$

$$c_{mn}^{(0)}(\bar{e}^{\hat{N}}, \bar{h}^{\hat{N}}) = 0,$$

$$d_{mn}^{(0)}(\bar{e}^{\hat{N}}, \bar{h}^{\hat{N}}) = 0, \quad m = \overline{-M,M}, \quad n = \overline{\max(1,|m|),N}. \quad (13)$$

Here, $\Psi_n(\bar{r}'(\theta))$ represents entire-boundary bases or sub-domain bases in the azimuthal plane $\varphi = \text{const}$, and $(\bar{\tau}_1, \bar{\tau}_2)$ are the unit vectors tangential to S.

In the standard formulation of the EBCM one uses global basis functions for the surface current density approximation. This system of functions is derived from the tangential components of the SVWF. Although these wavefunctions appear to provide a good approximation to the solution when S is smooth and "near" a spherical surface, they are disadvantageous when this is not the case. Furthermore, when the surface current densities have discontinuities in their continuity or any of their derivatives, straightforward application of global basis functions provides poor convergence to the solution. Wall [13] shows that the sub-domain basis or the local basis approximation of the surface current densities, when used with the null-field equations, result in slightly larger condition numbers for the matrix equations than would be obtained using global bases. In our computation we use linear and spline functions as local bases.

Once the surface current densities are determined a formal solution of the scattered field can be constructed in the form

$$\bar{E}_s^{\hat{N}}(\bar{r}) = \nabla \times \int_S \bar{e}^{\hat{N}}(\bar{r}')\, g(k_s|\bar{r} - \bar{r}'|)\,\mathrm{d}S' - \nabla \times \nabla \times \int_S \frac{1}{jk\varepsilon_s} \bar{h}^{\hat{N}}(\bar{r}')\, g(k_s|\bar{r} - \bar{r}'|)\,\mathrm{d}S',$$

$$r \in \Omega_s. \quad (14)$$

The far-field vector amplitude $\bar{F}_s^{\hat{N}}(\theta,\varphi)$ of the scattered field is calculated from

$$\lim_{r\to\infty} \bar{E}_S^{\hat{N}}(\bar{r}) = \bar{F}_S^{\hat{N}}(\theta,\varphi) \cdot \frac{\mathrm{e}^{jk_s r}}{r} \quad (15)$$

and the differential scattering cross-section from

$$\sigma_d^{\hat{N}} = \left|F_s^{\hat{N}}\right|^2. \quad (16)$$

The surface current densities which are solutions of the scattering problem guarantee the null-field condition of the total electric field within Ω_i. In the standard EBCM the surface current densities are obtained by imposing the null-field condition inside the maximal inscribed sphere. Due to the concept of analytic continuation the field will be zero through the entire interior volume. However, certain computations of the analytic continuation of fields confirm that the null-field condition deteriorates significantly in regions away from the inscribed sphere of expansion. Furthermore, the total electric field evaluated at the far ends of the dielectric object is characterized by poor convergence. This behavior is especially pronounced for dielectric volumes of large eccentricity, or when S has appreciable concavities, or geometrical singularities.

In order to compare different formulations of the EBCM we calculate a posterior error estimate, which is defined as the residual of the total electric field on different spherical surfaces enclosed in Ω_i

$$\delta E(w_p, R_p) = \frac{1}{R_p E_0} \left\| \bar{E}^{\hat{N}}(\bar{r}_p + w_p \bar{e}_3) \right\|_{S_{R_p}} = \frac{1}{R_p E_0} \left(\int_{S_{R_p}} |\bar{E}^{\hat{N}}(\bar{r}_p + w_p \bar{e}_3)|^2 \,\mathrm{d}S_{R_p} \right)^{1/2}. \quad (17)$$

Here, S_{R_p} is a spherical surface with radius R_p and having the center O_p at the point w_p on the z-axis.

3. Numerical results

For our numerical experiments we consider the scattering problem of a cone-sphere particle with a size parameter $k_s a = 5$, a refractive index $M = 1.5 + 0j$, and a half-cone angle $\alpha = 45°$. The geometry of the particle is illustrated in Fig. 1. The direction of propagation of the incident wave is along the Z-axis and the polarization direction makes an angle $\alpha_{\text{pol}} = 45°$ with the X-axis.

The residual of the electric field, computed using the standard EBCM is plotted in Fig. 2. As expected, the null-field condition deteriorates significantly outside the maximal inscribed sphere. The instability of the method is illustrated in Fig. 3, in which δE is computed in the vicinity of the tip for different numbers of unknowns. The

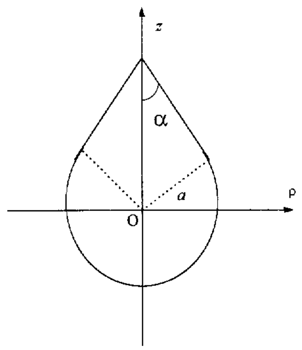

Fig. 1. Illustration of the cone-sphere geometry.

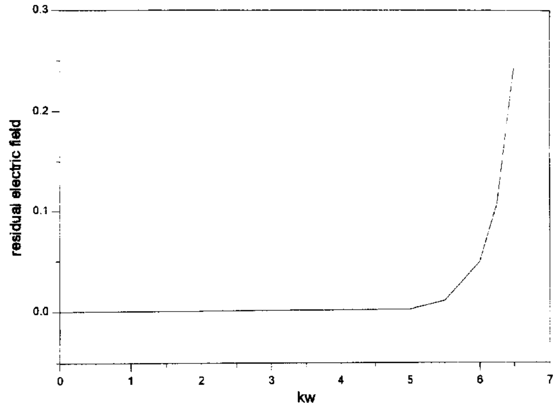

Fig. 2. Residual of the total electric field computed using the standard EBCM. The origin of the spherical surface is located at the point $k_s w$, which varies between 0.0 and 6.5. The radius of the spherical surface where the residual field is computed is $R_p = 0.9 R_{\max,p}$, where $R_{\max,p}$ is the radius of the maximal inscribed sphere having the origin at the point kw.

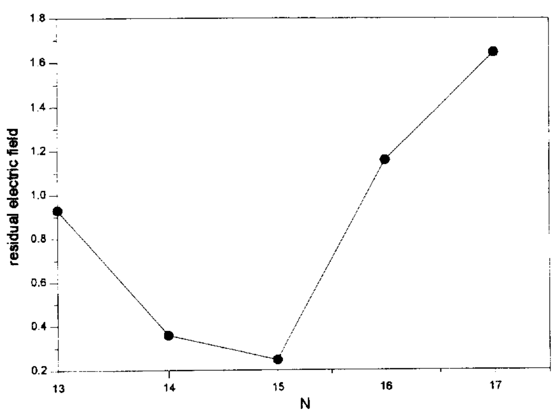

Fig. 3. Residual of the total electric field as a function on the number of unknowns N.

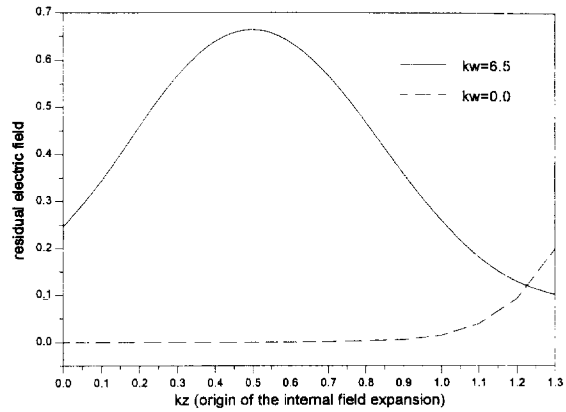

Fig. 4. Residual of the total electric field at the points $k_s w = 0.0$ and $k_s w = 6.5$ when the origin of the internal field expansion is shifted along the z-axis.

residual field increases with increasing N, and conversely the scattered field solution oscillates. However, the above procedure can be used to obtain the optimal solution which assures a minimal residual field. In Fig. 4 we show the values of the residual of the electric field, computed on the spherical surfaces with centers $k_s w_1 = 0.0$ and $k_s w_2 = 6.5$, when the origin of the internal field expansion is changed. This shifting procedure is temporary and of limited use, for though it does provide better residual field values in the regions around the shifted origin, the null-field condition in other parts of the dielectric object begins to deteriorate significantly as the inscribed sphere is shifted away from the center.

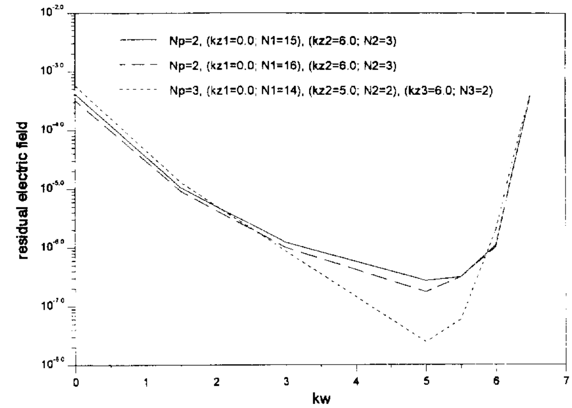

Fig. 5. Residual of the total electric field computed using the multiple multipole EBCM. The data correspond to the following arrangements of poles: (a) $N_p = 2$, $(k_s z_1 = 0,\ N(1) = 15)$, $(k_s z_2 = 6.0,\ N(2) = 3)$. (b) $N_p = 2$, $(k_s z_1 = 0,\ N(1) = 16)$, $(k_s z_2 = 6.0,\ N(2) = 3)$ and (c) $N_p = 3$, $(k_s z_1 = 0,\ N(1) = 14)$, $(k_s z_2 = 5.0,\ N(2) = 2)$, $(k_s z_3 = 6.0,\ N(3) = 2)$.

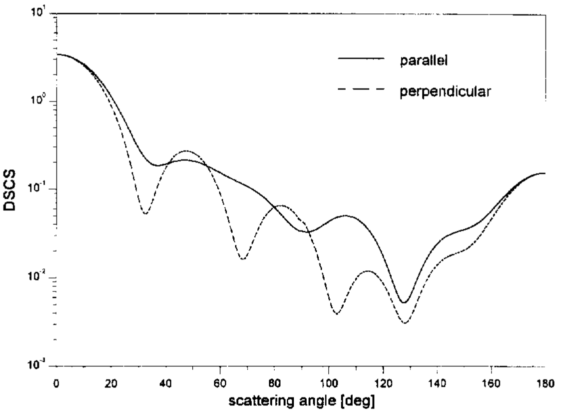

Fig. 6. The differential scattering cross section (DSCS) computed by using the multiple multipole EBCM.

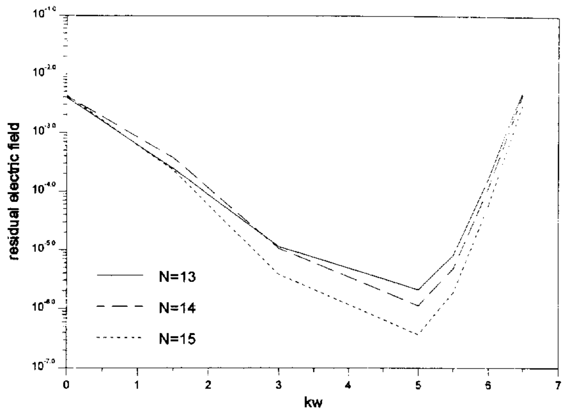

Fig. 8. Residual of the total electric field computed using the lowest-order multipole EBCM. The curves correspond to a number of sources $N = 13$, 14 and 15.

In Fig. 5 we show the residual electric field computed using the multiple multipole EBCM. The data correspond to the following arrangements of poles:

(a) $N_p = 2$, $(k_s z_1 = 0, N(1) = 15)$,

$(k_s z_2 = 6.0, N(2) = 3)$

(b) $N_p = 2$, $(k_s z_1 = 0, N(1) = 16)$,

$(k_s z_2 = 6.0, N(2) = 3)$,

(c) $N_p = 3$, $(k_s z_1 = 0, N(1) = 14)$,

$(k_s z_2 = 5.0, N(2) = 2)$, $(k_s z_3 = 6.0, N(3) = 2)$

In each case δE is smaller than 5×10^{-4} over the entire analysis domain. In order to compare the scattered field solutions computed with different theories we use this method as reference. Especially, the differential scattering cross section (DSCS) normalized by πa^2 will be evaluated in the azimuthal plane $\varphi = 0$. The DSCS computed using the multiple multipole EBCM is shown in Fig. 6. In Fig. 7 we illustrate the relative error of the DSCS obtained with the standard EBCM. The maximal value of the relative error is about 0.8%.

The residual of the total electric field computed with the lowest-order multipole EBCM is shown in Fig. 8. The curves correspond to a number of sources $N = 13$, 14 and 15. The maximal value of the residual field is about 5×10^{-3} for $N = 13$, and about 3×10^{-3} for $N = 15$. The relative error of the DSCS, illustrated in Fig. 9, is about 0.2%. In view of Figs. 8 and 9 one may conclude that this method is superior to the standard EBCM, but inferior to the multiple multipole EBCM. As it was shown in Ref. [12] the lowest-order multipole EBCM is suitable for scattering field calculations involving strong deformed particles, such as prolates and oblates with high aspect ratios.

In Fig. 10 we show the residual of the total electric field computed using the EBCM with sub-boundary bases. The curves correspond to a number of unknowns $N = 20$.

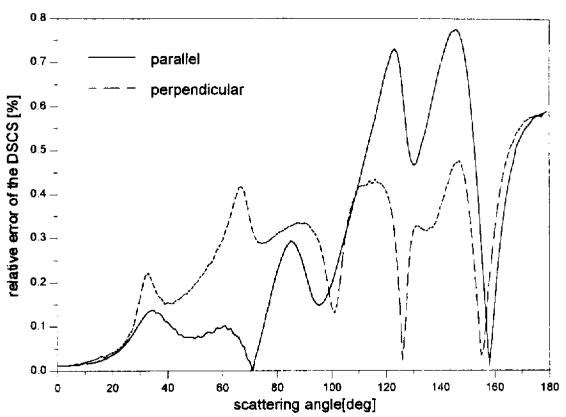

Fig. 7. Relative error of the DSCS computed using the standard EBCM.

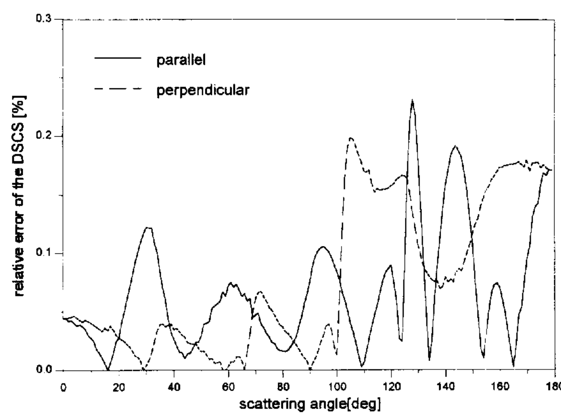

Fig. 9. Relative error of the DSCS computed using the lowest-order multipole EBCM.

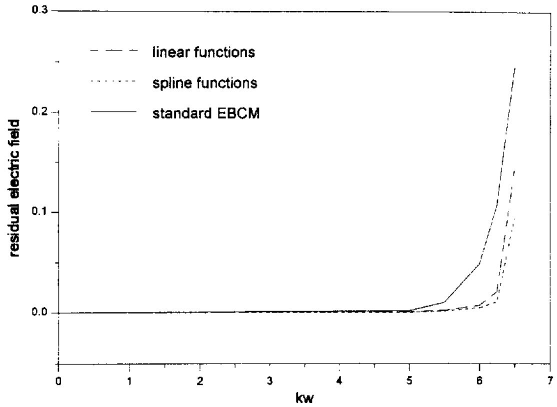

Fig. 10. Residual of the total electric field computed using the EBCM with sub-domain bases (linear and spline functions).

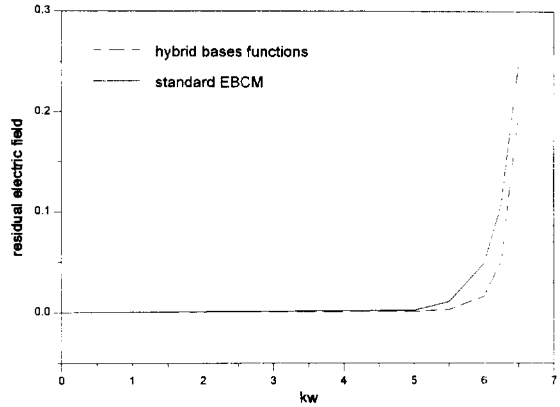

Fig. 12. Residual of the total electric field computed using hybrid basis functions (linear functions and the tangential components of the SVWF).

In accordance with the results given in Ref. [13] we found that local basis functions are more suitable to represent surface fields near the tip than the global basis functions. We mention here, that the evaluation of the residual of the electric field within Ω_i is not relevant, because the surface current densities computed according to (13) do not guarantee the null-field condition in Ω_S. In this case an additional criterion, consisting in evaluating the residual fields in Ω_S, must be used. However, the relative error of the DSCS, plotted in Fig. 11, indicates the superiority of the sub-boundary bases for the surface current density representation.

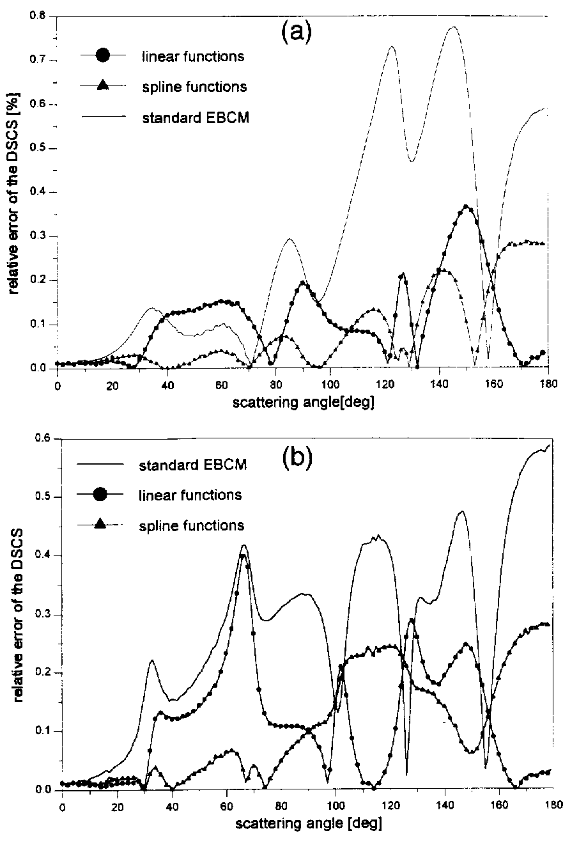

Fig. 11. Relative error of the DSCS computed using the EBCM with sub-domain bases (linear and spline functions): (a) parallel polarization, and (b) perpendicular polarization.

Conclusions concerning the computing time and the accuracy of the solution can be summarized as follows:

The standard EBCM needs less computing time but has an unstable convergence. The multiple multipole EBCM and the lowest-order multipole EBCM need rather lengthy computations (by a factor of 1.2–1.3 versus the standard EBCM), but give fast convergent and higher accurate solutions. The EBCM with sub-boundary bases requires lengthy computation (by a factor of 1.5–1.6 versus the standard EBCM), has slow convergence, but gives a more accurate solution than the standard EBCM.

An interesting approach is to combine sub-boundary bases with global-boundary bases. For example, the surface current densities can be expressed as a combination of linear basis functions (12) and the tangential components of SVWF with a single origin (6). The residual of the total electric field computed with this hybrid system of functions is shown in Fig. 12. In this case the use of linear basis functions ameliorates the behavior of the residual field in the vicinity of the tip.

4. Conclusions

A comparison of different formulations of the EBCM has been performed for a cone-sphere configuration. The residual of the total electric field on spherical surfaces enclosed in S was chosen as error criterion. The numerical results indicate that the multiple multipole EBCM and the lowest-order multipole EBCM are more accurate than the standard EBCM. Furthermore, the use of local basis functions (linear and spline functions) improves the stability of the classical approach. A hybrid approach, consisting in a combination of local basis functions and global basis functions (the tangential components of the SVWF with a single origin) was also investigated. Other hybrid methods which combine local basis functions with global basis functions given in Eqs. (8) or (10) are currently under study.

Acknowledgements

This research was supported by DFG (Deutsche Forschungsgemeinschaft).

References

[1] P.C. Waterman, J. Acoust. Soc. Am. 45 (1969) 1417.
[2] P.C. Waterman, Phys. Rev. D 3 (1971) 825.
[3] R.H.T. Bates, IEEE Trans Microwave Theory Tech. 23 (1975) 605.
[4] A. Lakhtakia, V.K. Varadan, V.V. Varadan, Appl. Optics 23 (1984) 3502.
[5] M.I. Mishchenko, L.D. Travis, Optics Comm. 109 (1994) 16–21.
[6] A. Bostrom, J. Acoust. Soc. Am. 76 (1984) 588.
[7] R.H. Hackman, J. Acoust. Soc. Am. 75 (1984) 35.
[8] M.F. Iskander, A. Lakhtakia, C.H. Durney, IEEE Trans Antennas Propag. 31 (1983) 317.
[9] R.H.T. Bates, D.J.N. Wall, Phil. Trans. R. Soc. Lond. A 287 (1977) 45.
[10] R.H. Hackman, J. Acoust. Soc. Am. 75 (1984) 35.
[11] A. Doicu, T. Wriedt, Formulations of the extended boundary condition method for incident Gaussian beams using multiple multipole expansions, J. Mod. Optics, accepted.
[12] A. Doicu, T. Wriedt, Optics Comm. 139 (1997) 85.
[13] D.J.N. Wall, in: Acoustic, Electromagnetic and Elastic Wave Scattering, Eds. V.K. Varadan and V.V. Varadan (Pergamon, New York, 1980) pp. 269–286.
[14] R.H.T. Bates, in: Acoustic, Electromagnetic and Elastic Wave Scattering, Eds. V.K. Varadan and V.V. Varadan (Pergamon, New York, 1980) pp. 21–31.
[15] P.W. Barber, S.C. Hill, Light scattering by particles: computational methods (World Scientific, Singapore, 1990).
[16] M.I. Mishchenko, L.D. Travis, A. Macke, Appl. Optics 24 (1996) 4927.
[17] C. Hafner, The Generalized Multipole Technique for Computational Electromagnetics (Artech, Norwood, MA, 1990).
[18] A.C. Ludwig, Comput. Phys. Comm. 68 (1991) 306.