A Review of Elastic Light Scattering Theories

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Abstract

Mie scattering is an important tool for diagnosing microparticles or aerosol particles in technical or natural environments. Mie theory is restricted to spherical, homogeneous, isotropic and non-magnetic particles in a non-absorbing medium. However, as microparticles are hardly ever spherical or homogeneous, there is much interest in more advanced scattering theories. During recent decades, scattering methods for non-spherical and non-homogeneous particles have been developed and even some computer codes are readily available. Extension of Mie theory covers coated spheres, stratified spheres and clustered spheres. For homogeneous non-spherical particles such as spheroids, ellipsoids and finite cylinders, surface discretization methods have been developed. Scattering by inhomogeneous particles may be computed by volume discretization methods.

1 Introduction

In the field of optical particle sizing, to relate optical measurements with the properties of the particles to be characterized one must be able to describe the scattering of light by particles. Light scattering theories are needed to analyse optical particle counters such as phase Doppler anemometry (PDA), visibility- or intensity-based counters, laser diffraction instruments and light extinction instruments. Scattering computations help in understanding new physical phenomena or in designing new particle diagnostics systems for the identification of variations in particle optical properties or particle shape. Furthermore, computation of light scattering by particles plays an enormous role not only in optical particle sizing but also in astronomy, optical oceanography, photographic science, meteorology and coatings technology, to name but a few. Similar electromagnetic modelling methods are needed to investigate microwave scattering by raindrops and ice crystals or to analyse electromagnetic interference problems.

Until recently mainly, Mie theory was applied to analyse optical particle sizers. Mie theory is restricted to plane wave scattering by a spherical homogeneous, isotropic and non-magnetic particle in a non-absorbing medium. However, as particles of interest are hardly ever spherical or homogeneous, there is much interest in more advanced scattering theories that are not that much restricted. In recent decades, scattering theories of non-spherical and inhomogeneous particles have been developed and even some computer codes are readily available [1]. These developments will be reviewed here, together with references to the latest publications to enable the reader to dig deeper into the relevant literature. A collection of papers related to the subject of this review can be found in the SPIE Milestone volume ‘‘Selected Papers on Light Scattering’’ edited by *Kerker* [2] and in the feature edition on ‘‘Scattering by Three Dimensional Objects’’ edited by *Barber*, *Miller* and *Sarkar* [3]. Papers based on a Workshop of ‘‘Scattering by Non-spherical Particles’’ are printed in a special edition [4]. Papers presenting an electrical engineering view on scattering have been assembled by *Hanson* [5] and by *Ross Stone* [6] and another one by *Miller*, *Medgyesi-Mitschang* and *Newman* [7]. Similar reviews from which some information has been drawn have been published by *Bohren* [8], on scattering by particles, and by *Perez* [9] and *Volakis* and *Kempel* [10], on computational methods in electromagnetics.

For the purpose of this review, the different scattering methods will be classified under the following headings: analytical methods, surface-based methods, volume-based methods and multiple scattering. Analytical methods are based on a separation of variables approach. In surface-based methods, the boundary conditions are enforced on the surface of the scattering particle and only this surface is discretised. With volume-based methods, the volume of the particle and, with some methods, also part of the surrounding medium is discretised. Within the context of this review, multiple scattering means scattering by a few neighbouring particles. Both well established and ‘‘new’’ theories are covered to help the reader to consider different methods for specific particle sizing problems.

2 Analytical Methods

2.1 Extensions of Mie Theory

The simplest representation of a scattering particle is a homogeneous isotropic sphere. Scattering by such a sphere is referred to as Mie theory, although *Gustav Mie* (1868–1957) was not the first to formulate this electromagnetic scattering problem. Before him *Alfred Clebsch* (1833–72) and *Ludvig Lorenz* (1829–91) contributed to this problem. The interesting early history of light scattering has been reviewed by *Logan* [11]. The first modern outline of the Mie theory in terms of spherical vector wavefunctions is given in the classical book by *Stratton* [12]. Mie theory is a separation of variables approach which gives an analytical equation for the Mie coefficients.

As Mie theory is restricted to spherical homogeneous spheres, there are many extensions of this theory covering different aspects. Some relevant aspects will be described in the following. A scattering theory for magnetic spheres can easily be formulated

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[13]. This may be relevant for scattering at infrared or microwave frequencies.

The scattering theory of coated dielectric spheres was first derived by *Aden* and *Kerker* [13]. An advanced algorithm was given by *Toon* and *Ackerman* [15]. An algorithm for a sphere having two coatings was given by *Kaiser* [16]. This has been used to compute the internal field of a particle at resonance. Such algorithms may help in identifying water droplets collecting dust or soot on the outer surface [17]. Different scattering particles may be modelled in this way, e.g. water-coated soot particles, hydrological particles coated with biological material and micro-encapsulated material. *De Pieri* [18] measured the differential light scattering patterns of single bacterial spores and matched this pattern to theoretical patterns computed by a core shell model. In this way the inner (or protoplast) and outer (or integument) refractive indices and the inner and integument thickness of the spore were found. A three-layered sphere model for phytoplankton has been used too by *Kitchen* and *Zaneveld* [19] to reproduce measured volume scattering functions of natural seawater.

A stratified particle having a radial refractive index gradient is treated was *Bhandari* [20], and more recently by *Wu* and *Wang* [21] and *Kai* and *Massoli* [22]. These scattering theories will be of interest when investigating the radial gradient of the refractive index of droplets resulting from vaporization in combustion sprays. Spheres which are eccentrically multilayered are treated by analytical solutions by using the addition theorem for spherical vector wavefunctions [23]. Based on such an approach, a FORTRAN code for a sphere with a non-concentric spherical inclusion has been published recently by *Ngo, Videen* and *Chýlek* [24].

A scattering theory for a chiral sphere has been presented by *Bohren* [25]. A chiral sphere has different refractive indices for a right- and a left-polarized incident wave. Chirality is a specific property of biologically derived molecules. A chiral sphere may thus represent biological aerosols such as pollen, bacteria and spores.

Another derivation from a Mie sphere is a slightly non-spherical particle. This may be treated by a first-order perturbation approach [26]. In this case the assumptions are that the particle is homogeneous and that the deviations from sphericity are small and smooth, such as a droplet distorted by a fluid flow. There is also an extension of Mie theory to an anisotropic spherical shell [27] which is an appropriate model to study light scattering by a variety of biological systems.

A further extension of Mie theory which is of great interest in optical particle sizing is scattering of a sphere excited by a laser beam having a Gaussian intensity distribution. The beam may be expanded into spherical vector wavefunctions by computing beam shape coefficients [28] in the framework of the generalized Lorenz-Mie theory (GLMT) or into a spectrum of plane waves [29–31]. Different methods to compute the beam shape coefficients have been developed. A rigorous approach is based on surface integration [32]. The coefficients of a Gaussian beam can also be computed by a finite series for on-axis particle positions [33]. The localized approximation of the beam shape coefficients leads to the fastest algorithm, as has been demonstrated by *Lock* [34]. Based on such methods, the Gaussian beam effect which causes erroneous particle size measurements in PDA can be analysed and corrected [28]. This method can also be applied to computation of the morphological resonances induced by off-axis illumination of a sphere by a focused laser beam [35].

The GLMT has recently been adapted to shaped beam scattering by a coated sphere [36] and a multilayered sphere [37]. This may be of interest when diagnosing fuel droplets in a combustion spay which exhibit a radial temperature gradient and therefore a gradient of the refractive index.

2.2 Spheroidal Expansion

Maxwell's equations are also separable in a spheroidal coordinate system. Therefore, a separation of variables approach can also be applied. In this way, scattering by a spheroid (a prolate or oblate ellipse of revolution) can be considered. Besides a sphere, a spheroid is considered to be a simple model of an atmospheric aerosol particle. The method is similar to Mie theory. The surface of the scattering spheroid coincides with one of the coordinate surfaces. The incident, transmitted and scattered fields are expanded in terms of spheroidal wavefunctions. The coefficients of the scattered field are then found by imposing the boundary conditions on the particle surface. The theory was presented in a much-cited paper by *Asano* and *Yamamoto* [38], and results were given for aspect ratios (ratio of minor axis to major axis) a/b of 5 and particle size parameters $\alpha = 2\,\pi a/\lambda$ of up to 35 [39]. Another spheroidal expansion method was given by *Voshchinnikov* and *Farafonov* [40], and results for extremely oblate and prolate spheroids having an aspect ratio of up to 100 were given. A further implementation of the method of separation coordinate system applied to oblate spheroids having an aspect ratio of 0.1 was presented by *Kurtz* and *Salib* [41], who found *Asano's* method to be unacceptably inaccurate in this extreme oblate case. They were interested in computation scattering by disc-shaped silver halide crystals in gelatin to investigate scattering of photographic emulsions. The spheroidal expansion method can also be applied to coated spheroids [42, 43], represented by a silicate particle with a non-absorbing cladding. For a spheroid illuminated by a shaped beam, *Barton* [44] applied an electromagnetic approach generalizing *Asano's* work.

3 Surface-Based Methods

3.1 Point Matching Method

The point matching technique is a differential equation formulation which was developed to compute microwave scattering by spheroidal raindrops [45, 46]. The formulation of the technique closely follows Mie theory. Similarly to Mie theory, the scattered field and transmitted field are expanded into terms of spherical vector wavefunctions (also called multipoles) $\boldsymbol{M}_n$, $\boldsymbol{N}_n$. For example, the expansion of the scattered electric field $\mathbf{E}^s$ as a function of the position vector into outgoing spherical vector wavefunctions $\boldsymbol{M}_n^3$, $\boldsymbol{N}_n^3$ is given by

$$\boldsymbol{E}^s(\boldsymbol{r}) = \sum_{n=1}^{N} D_n \left[a_n^s \boldsymbol{M}_n^3(k\boldsymbol{r}) + b_n^s \boldsymbol{N}_n^3(k\mathbf{r}) \right]. \qquad (1)$$

The expansion coefficients of the scattered field a_n^s and b_n^s are found by satisfying the boundary conditions at the surface of the scatterer by combining point matching with a least-squares fitting procedure. the boundary conditions require the continuity of the tangential components of the electric and magnetic fields across the surface of the scatterer. The incident plane wave is expanded into a complex Fourier series in the azimuthal angle. Because an axially asymmetric scatterer is considered, the boundary condition can be enforced independently for each term of the Fourier series. The concept of the point matching method is simple and it is suitable for rotationally symmetric scatterers. The matrix to be inverted is larger than the matrix needed by the T-matrix method for the same scatterer. The method is considered to have uncertain convergence and accuracy and needs much computer time [47]. The point matching method has recently been applied to compute microwave scattering by oblate spheroidal hydrometeors up to a size parameter of 9.42, expressed as the equivolume radius [48]. The point matching method has also been adapted to coated axisymmetric objects [49] and scattering computations have been compared with EBCM results. Because this method does not use the translation addition theorem for spherical vector wavefunctions, much less analytical and programming effort is needed to implement this method. For all cases of microwave scattering by hydrometeors considered, good agreement was claimed when comparing results from the different methods. Plots of the normalized differential scattering cross-section patterns were given for spheroid with a size parameter $k_a = 12.9952$ and an aspect ratio a/b = 1.88.

Apart from the point matching method there are other differential equation approaches. Recently, the method of lines [50] has been adapted for scattering computations of non-spherical axisymmetric scatterers. This method has been named the discretised Mie formalism [51]. By applying a difference operator, a system of coupled ordinary differential equations results. These equations are decoupled by a suitable transformation, such that the equations can be solved analytically as in Mie theory. In this way the radiation condition is analytically incorporated.

Another differential formulation of electromagnetic scattering by rotationally symmetric scatterers was presented by *Tosun* [52]. This method is also based on the field representation by spherical vector wavefunctions. An inhomogeneous scatterer can be considered by having a position-independent expansion coefficient. The unknown expansion coefficients of the far field are found by matrix inversion. As an example, *Tosun* [52] computed scattering by a Luneburg lens.

3.2 T-Matrix Method

The T-matrix method is another well known technique which has found a wide range of applications because a code for a conducting scatterer was published very early [53]. A disk with a FORTRAN code for dielectric bodies of rotation is included with the book by *Barber* and *Hill* [54]. The T-matrix method is also called the null field method or extended boundary condition method (EBCM). It is based on a series of papers by *Waterman* [55]. An early collection of conference papers on this method was edited by *Varadan* and *Varadan* [56].

With this method, the incident transmitted and scattered field is expanded into a series of spherical vector wavefunctions as shown for the scattered field:

$$\boldsymbol{E}^s(\boldsymbol{r}) = \sum_{n=1}^{N} D_n \left[a_n^s \boldsymbol{M}_n^3(k\boldsymbol{r}) + b_n^s \boldsymbol{N}_n^3(k\boldsymbol{r}) \right]$$

$$\begin{bmatrix} a^s \\ b^s \end{bmatrix} = -T \begin{bmatrix} a^i \\ b^i \end{bmatrix}. \tag{2}$$

The expansion coefficients of the scattered field a^s, b^s are related to the coefficients of the incident field a^i, b^i by the T-matrix (transition matrix).

The elements of the T-matrix are obtained by numerical integration. For an arbitrarily shaped particle a surface integral has to be computed. As this is computationally expensive, most implementations of the methods are restricted to axisymmetric scatterers. In this case line integrals have to be computed. Nevertheless, there are some papers in which the T-matrix method was applied to arbitrarily shaped scatterers [57, 58]. *Wriedt* and *Doicu* [58] presented computational examples with results of scattering by a dielectric cube of size parameter 2 and by a dielectric spheroid of size parameter 20. Scattering by a dielectric cube was computed by both the extended boundary condition method and the extended boundary condition method with discrete sources.

It is easy to extend the T-matrix method to coated spheroids [59, 60] in order to model water-coated ice particles in the atmosphere. An incident Gaussian beam can also easily be included in the theory [29, 31]. *Mishchenko* and *Travis* [61] demonstrated that using extended precision instead of double precision variables helps to improve the convergence of the method up to particle size parameters of 100 (equal surface area sphere size parameter). This corresponds to the author's experience. A review of the current status of the T-matrix approach has been published by *Mishchenko, Travis* and *Mackowski* [62]. They concluded that recent improvements make the method applicable to particles with size parameters well exceeding 50.

3.3 Generalized Multipole Technique

The generalized multipole technique (GMT) is a relatively new and fast advancing method which has been developed by different research groups. *Ludwig* [63] coined the term generalized multipole technique for this spectrum of methods. In Mie theory and in the T-matrix method, the fields inside and outside a scatterer are expanded by a set of spherical multipoles having their origin at the centre of the sphere. With the GMT method many origins are applied for multipole expansion. The coefficients of these expansions are the unknown values to be determined by applying the boundary conditions on the particle surface. The coefficients may be found by point matching, that is, fulfilling the boundary conditions at a discrete number of surface points, or fulfilling the boundary conditions in a least-squares sense, or by a surface integral similar to the extended boundary condition method [64]. Not only multiple spherical multipoles can be used for field expansion, other ‘‘equivalent sources’’ are also possible. The ‘‘equivalent sources’’ may be of any type, as long as they are solutions of the wave equation. Spherical waves, dipoles and Gabor functions have been applied as expansion functions. Therefore, other names for similar concepts have been given, e.g. multiple multipole method (MMP) [65], discrete sources method (DSM) [66], fictitious sources method [67] or *Yasuura* method [68]. The MMP method has been developed for arbitrary three-dimensional electromagnetic problems. The FORTRAN code supplied with the book by *Hafner* and *Bomholt* [65]can be easily adapted to compute scattering by various three-dimensional objects. The multipoles describing the internal field are located outside the scattering particle and the poles expanding the external field are positioned within the particle. In DSM multipole sources are distributed on the axis of the axisymmetric scatterer. As the incident field is given by numerical values of the electric and magnetic field on the particle surface, the GMT methods can easily be extended to Gaussian laser beam scattering [69]. No expansion of the Gaussian beam into spherical wavefunctions or into plane waves is needed.

3.4 Methods of Moments

The methods of moments (MoM) is a surface integral equation method which has a long history in computational electromagnetics.

Its origin dates back to the work of *Harrington* [70]. The method of moments solves the functional equation

$$L\,\boldsymbol{F} = \boldsymbol{g} \tag{3}$$

representing the boundary condition of the tangential field components on the particle surface. L is a linear operator giving the electric and magnetic field values due to the surface currents, vector $\boldsymbol{g}$ encloses the known incident electric and magnetic field values and $\boldsymbol{F}$ is an unknown vector of electric and magnetic surface currents. $\boldsymbol{F}$ is expanded in a linear combination of basis functions $\boldsymbol{f}_n$:

$$\boldsymbol{F} = \sum_{n=1}^{N} \alpha_n \boldsymbol{f}_n \tag{4}$$

where α_n are unknown coefficients. Substituting this expansion of $\boldsymbol{F}$ into the functional equation gives

$$\sum_{n=1}^{N} \alpha_n\, L\, \boldsymbol{f}_n = \boldsymbol{g}. \tag{5}$$

This equation is solved by multiplying it with a set of testing functions $\boldsymbol{w}_m$ and forming an inner product with both sides of this equation:

$$\sum_{n=1}^{N} \alpha_n \int \boldsymbol{w}_m\, L\, \boldsymbol{f}_n = \int \boldsymbol{w}_m \boldsymbol{g}. \tag{6}$$

This set of equations can be written in matrix form:

$$[C]\boldsymbol{x} = \boldsymbol{g}. \tag{7}$$

The column vector $\boldsymbol{g}$ contains the incident field values and the column vector $\boldsymbol{x}$ contains the unknown coefficients to be determined. Hence, the original integral equation is transformed into a set of linear equations. When solving electric or magnetic field integral equations, α are the coefficients of the induced surface currents. From this, scattered fields can be computed. A review of this method was given by *Harrington* [71]. *Kishk* and *Abouzahra* [72] used MoM to compute scattering by various inhomogeneous objects: toroids divided into six regions, conducting bicones and conducting hemispheres. *Medgyesi-Methang*, *Putnam* and *Gedera* [73] demonstrated that their MoM code, developed for radar cross-section computations, can be applied to study scattering by dielectric spheres, hollow spheres and spheroidal shells with spherical voids or a conducting core.

4 Volume-based Methods

4.1 Finite Difference Time Domain

The finite difference time domain (FDTD) method is an electromagnetic modelling technique which has recently become very popular in electromagnetics [74], but it is also very suitable for scattering computations of non-spherical and inhomogeneous particles. With the FDTD method an entire volume including the scatterer is discretised. The basic element of this discretisation is the Yee cell, named after *K. S. Yee* [101] who originally proposed this method. The Yee cell is the basic element of an interlocked grid with the electric field $\boldsymbol{E}$ representing an unknown on the edges of one grid and the magnetic field $\boldsymbol{H}$ the unknowns on the other, corresponding to the faces of the grid. The differential form of Maxwell's equations are solved directly. A difference approximation is applied to evaluate the space and time derivatives of the field. The fields are computed by a time marching scheme:

$$\boldsymbol{H}^{n+1/2}_{x(ijk)} = \boldsymbol{H}^{n-1/2}_{x(ijk)} + \frac{\Delta t}{\mu \Delta z}\left(\boldsymbol{E}^{n}_{y(ijk)} - \boldsymbol{E}^{n}_{y(ijk-1)}\right) - \frac{\Delta t}{\mu \Delta y}\left(\boldsymbol{E}^{n}_{z(ijk)} - \boldsymbol{E}^{n}_{z(ij-1k)}\right). \tag{8}$$

The unknown value of the magnetic field $\boldsymbol{H}^{n+1/2}$ depends on the old value of the magnetic field $\boldsymbol{H}^{n-1/2}$ and on the old value of the electric field $\boldsymbol{E}^{n}$ on either side of the magnetic field point (i,j,k) in space. The time step is denoted by Δt and the grid spacing by Δx, Δy, Δz. μ is the magnetic permeability. A similar difference equation is used to find the unknown electric field from the magnetic field. By alternating these two computations at each time step, the field propagates through the whole volume. This time marching scheme is applied until a steady-state solution is obtained. Each grid point of the volume may have different values of permittivity so that an inhomogeneous scatterer can be computed. For simulation of scattering special absorbing boundary conditions are needed such that the wave is not reflected at the open boundary of the discretised volume. A near-field to far-field transformation is needed to compute the scattered far-field from the near-field values of the computational volume. The FDTD method requires not only the scatterer to be discretised but also part of the near-field around it. The grid distance must be smaller than the wavelength of the incident wave. The FDTD method has recently been used to compute microwave [75] and light scattering [76] by ice crystals. FDTD results were compared with Mie theory. There was good agreement between FDTD and Mie results for a size parameter of 15 when a grid size of $\lambda/20$ was used [76]. Because only nearby grid points couple to each other, in contrast to MoM, no matrix has to be stored and inverted. This is considered a major advantage of this method. However, care must be taken because time domain methods may be unstable and the time steps must be chosen such that certain stability criteria are ensured.

4.2 Transmission Line Matrix

Similarly to the FDTD method, but less well known, is the transmission line matrix (TLM) method [102]. In TLM the scatterer and part of the surrounding medium are discretised into TLM cells. These cells represent elementary multiports which are connected to neighbouring cells by transmission lines, thus forming a three-dimensional network of transmission lines. The inductive, capacitive and resistive properties of the transmission lines model the magnetic, dielectric and absorptive parameters of the scattering particle. In the TLM method, the electromagnetic field is represented by wave amplitudes of the wave propagating on the transmission lines. The magnetic field is analogous to the currents and the electric field is analogous to the voltages on the transmission lines. As with FDTD, complex and even inhomogeneous scatterers can be modelled but the computational demands are severe because the particle has to be discretised with a fine grid. A special near- to far-field transformation is needed to compute the far-field which is of most interest in optics. The far-field components are computed from the tangential electric and magnetic field on a closed surface enclosing the scattering particle. Recently, the radar cross-section of a dielectric cube having a cube side length of 1.4 wavelengths has been computed by the TLM method [77].

4.3 Volume Integral Equation

The electric volume integral equation (VIE) may be solved by two similar methods to compute scattering by an inhomogeneous scatterer [78]. The first approach is based on the method of moments (MoM) [79], the second on the coupled dipole method (CDM) [80] or discrete dipole approximation (DDA) [81]. The MoM is an actual field formulation, whereas the coupled dipole method is based on the concept of excited fields [78]. The DDA method was originally proposed by *Purcel* and *Pennypacker* [82]. In DDA an arbitrarily shaped particle is treated as a three-dimensional assembly of dipoles on a cubic grid. Each dipole cell is assigned a complex polarisability α which can be computed from the complex refractive index of the bulk material and the number of dipoles in a unit volume. This polarisability causes an oscillating dipole moment or polarization $\boldsymbol{P}_i$ at each cell, depending on the total electric field at the respective position:

$$\boldsymbol{P}_i = \alpha_i \boldsymbol{E}_i. \tag{9}$$

The total field $\boldsymbol{E}_i$ is the sum of the incident field $\boldsymbol{E}_{inc,i}$ and a contribution from all other dipoles:

$$\boldsymbol{E}_{other,i} = -\sum_{i \neq j} \mathbf{A}_{ij} \mathbf{P}_j \tag{10}$$

$$(\alpha_1)^{-1} \boldsymbol{P}_j + \sum_{i \neq j} \mathbf{A}_{ij} \boldsymbol{P}_j = \mathbf{E}_{inc,j}. \tag{11}$$

The matrix $\boldsymbol{A}_{i,j}$ includes interaction of all dipoles depending on the distance of the dipoles, thus a full dense matrix results. The incident field is a plane wave. This equation can be solved by iteration. Beginning with an initial guess of $\mathbf{P}_i$, a convergent solution can be obtained.

The MoM has been used to compute back-scattering by cubes having a size parameter 4a/λ of 13 [79] and of 8.19 [83]. The DDA has been applied to compute scattering by aggregated spheres [84]. The standard check of the accuracy of any DDA model is a comparison with results from Mie theory. This has been done for size parameters of up to 4.926, represented by 2320 pseudo-spheres. The agreement between DDA and Mie theory is very good. Over the years the discrete dipole approximation has become a powerful method for computing electromagnetic scattering by arbitrarily shaped bodies as shown in the review of this technique by *Draine* and *Flatau* [85]. A recent improvement of this technique has been introduced by *Piller* [86], applying concepts from sampling theory. By introducing a filtered Green tensor, the method produces reliable results for extremely rough discretization grids such as 2.22 meshes per wavelength.

4.4 Finite Element Method

Finite element (FE) methods, which are well known to mechanical engineers, have also found their way into electromagnetics, normally as a formulation in the frequency domain. They permit one to handle complex scattering shapes. Whereas in the FTDT and TLM methods a scattering particle is discretised on a cubic grid, which can lead to staircase approximations of the shape of the scattering particle, in FE methods the particle shape can be discretised by using a variety of elements of different shapes. Three-dimensional shapes may be the hexahedron, which has eight nodes and twelve edges, and the tetrahedron, having four nodes and six edges. The object of FE analysis is to find the electromagnetic field values at the nodes or alternatively at the edges of these shapes. The sparsity of the resulting matrix and its geometrical flexibility are considered to be a major advantage of FE methods over VIE methods [87]. An FE method has recently been applied to compute light absorption by a coated particle [88]. The results compared well with Kerker's analytical solution for a water-coated sphere with a carbon core.

5 Multiple Scattering

In this section, multiple scattering by a few neighbouring particles is reviewed. A simple problem of multiple scattering is a cluster of two spheres. This problem was tackled in the classical paper of *Bruning* and *Lo* [89], although there appear to be some typographical errors in the equations. The theory is based on a field expansion into spherical vector wavefunctions as in Mie theory, and makes use of the translation addition theorem for such expansion functions. By using the translation addition theorem the field scattered by one sphere is expanded into spherical multipoles having their origin at the centre of the other sphere. This method has been applied to show that enhanced back-scattering can be produced by clusters composed of two scattering particles [90, 91]. The method can also be extended to the T-matrix method to compute scattering by two spheroids [59]. It has also been applied to scattering of a system of two spheroids on the basis of the translational addition theorem for vector spheroidal wavefunctions [92]. *Hamid* investigated the possibility of modelling the scattering from a dielectric spheroid by a system of three dielectric spheres [93].

Apart from methods based on the translation addition theorem, other surface discretisation methods are also suitable for computing multiple scattering, e.g. the discrete sources method (DSM) has been used to compute microwave scattering by two axisymmetric raindrops [94]. The MMP method [65] can also easily be used to simulate multiple scattering problems. If also the Gaussian laser beam is accounted for, complex particle sizing problems can be simulated [95]. Another method for simulating electromagnetic scattering from clusters of dielectric particles is the boundary element method [96]. Computational results were given for a system of three spheres having a size parameter k_a of 2.5. The method of moments has also been applied to agglomerated spheres [73]. Volume discretisation methods can be applied to multiple scattering problems too. *Flatau* et al. [97] compared the DDA method with the modal analysis method for the case of scattering by two spheres. To test DDA they computed scattering by two dielectric spheres, each having a size parameter k_a of 30. The comparison showed excellent agreement between the two methods.

6 Conclusion

Each theory reviewed has its own range of applicability, which mainly depends on the particle shape, its composition and refractive index and its size relative to the wavelength of the incident wave. Consequently, a single theory will not cover all possible scattering particles and problems of application.

When choosing a scattering method for a specific optical particle sizing problem, one should also consider the amount of analytical and programming efforts needed for its implementation. It is advisable to decide on a scattering theory with which one is familiar, rather than a more complex theory which is less understood. One should also consider the demands of the respective method in terms of computer resources, computer memory and execution time, in addition to the parameters of the method in question that govern the accuracy of the final computational results. A surface-based method will need less computer memory than a volume-based method for the same size of the scatterer. Hence with surface-based methods scattering by larger particles can be computed.

There are only a few papers comparing more than just two scattering methods. An example is given by *Hovenier* et al. [98], who compared three different scattering codes: T-matrix, a separation of variables method for spheroids (SVM) and a DDA programme. Scattering by a prolate and an oblate spheroid and a finite cylinder was computed by these three methods. The scatterers had an equal-volume sphere size parameter of 5 and a refractive index of $1.5-0.1i$. The T-matrix method and the SVM showed perfect agreement, whereas there was poorer agreement with DDA, but the general curves were reproduced by the latter.

Taking an electrical engineering viewpoint, different electromagnetic codes were compared by *Cooper*, *Hombach* and *Schiavoni* [99]. They compared two surface-based codes (MoM and MMP) and two volume-based time-domain codes (finite integration technique (FIT) [100] and FDTD) with the analytical Mie solution. A sphere and a coated sphere having a size parameter of 1.887 were adapted as a simple model of a human head, and the internal fields were computed with an incident plane wave or a dipole excitation that represented the antenna of a hand-held telephone. The MMP code gave the smallest number of unknowns, with the MoM needing ten times as much. This method also required the least CPU time. There is practically no difference from the internal field computed with an incident plane wave. There are many more differences between results from different methods with dipole excitation, but this seems to be of less interest to this review.

A comparison of different light scattering theories for a specific particle sizing problem is in progress and will be published in a subsequent paper.

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8 Symbols and Abbreviations

a	semimajor axis of spheroid
a_n	expansion coefficient
$A_{i,j}$	interaction matrix
b	semiminor axis of spheroid
b_n	expansion coefficient
C	matrix
D_n	normalisation constant
f_n	basis functions
$\boldsymbol{g}$	vector of incident electric and magnetic field values
$\boldsymbol{F}$	vector of electric and magnetic surface currents
k	propagation constant
L	linear operator
$\boldsymbol{M}_n$	spherical vector wavefunctions
$\boldsymbol{N}_n$	spherical vector wavefunctions
$\boldsymbol{P}_i$	polarization
$\boldsymbol{r}$	position vector
Δt	time step
T	transition matrix
w_m	testing functions
Δx, Δy, Δz	grid spacing
α_i	complex polarisability
α_n	expansion coefficient
ϵ	permittivity
λ	wavelength
μ	permeability

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