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Reconstruction of velocity-diameter relations obtained from PDA measurements during atomization of inhomogeneous liquids

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ABSTRACT

If the characteristics of process fluids differ from the ideal conditions which are assumed for PDA measurements, bias effects occur - especially in diameter determination. In most cases the biases result in a significant spread of the measured phasedifferences around the phasedifference-diameter relation, which is calculated by a light scattering program for the given PDA-setup. As a consequence the boarders of size distributions are flattened out. Furthermore the bias influences in the same manner the velocity-diameter correlation, which is often approximated by an 'Empirical Regression' of the mean velocities of every size bin $V_{1,0}(D_i)$. Without any further reflection the evaluation of the measured data leads to results far from reality.

The bias effects can be taken into consideration by application of more sophisticated postprocessing procedures. In this presentation a mathematical model is introduced which can be applied to get corrected information about the diameter and the velocity of the droplets. The validity of the model is proofed by the comparison of PDA measured data of an inhomogeneous media (undiluted condensed milk) and of an homogeneous (ethylene glycol) with identical physical properties.

NOTATION

C	Correction factor	q	density distribution
D	Diameter	q_0	number density distribution
DV	velocity-diameter correlation (-matrix)	TM	Transfer matrix
n	number of iteration steps	V	Velocity
N	Total number of measured size bins	$V_{1,0}$	Mean velocity

Subscripts

i,j	row and columns of a matrix

INTRODUCTION

In many industrial applications atomization of liquids is an important unit operation. The structure of an enamelled surface, the efficiency of fuel combustion or the quality of spray dried products are all affected by the velocity and size distribution of the droplets. Optimization of these two parameters improves the characteristics of the final product.

Phase-Doppler-Anemometry (PDA) is well established and enables the simultaneous measurement of both parameters: velocity and diameter of droplets. The measuring accuracy of the PDA strongly depends on the optical properties of the disintegrated phase (homogeneous - inhomogeneous), on the particle characteristics (spherical - ellipsoid, smooth - rough) and finally on the quality of the measured Doppler bursts (signal-to-noise-ratio). Under ideal conditions PDA has been applied successfully to investigate different spraying processes like atomization of fuel and water.

A typical application in which difficulties arise for the PDA measurements is the spray drying of optically inhomogeneous media [3]. It was already demonstrated that the measuring accuracy in diameter determination decreases with increasing concentration of the inhomogeneities whereas the accuracy in velocity determination stays nearly constant [5]. The less measuring accuracy can be mathematically interpreted as a significant Gaussian spread of the measured phasedifferences around the phasedifference-diameter relation, which is calculated by a light scattering program for the given PDA-setup. Though the application of more sophisticated mathematical algorithms allows the PDA user to get correct informations about the true diameter distribution [6]. Recent experiments make clear that a lower measuring accuracy in diameter determination influences in the same manner the velocity-diameter correlation, which is often approximated by an 'Empirical Regression' of the mean velocities of every size bin $V_{1,0}(D_i)$.

HOMOGENOUS AND INHOMOGENEOUS LIQUIDS MEASURED BY PDA

To get comparable PDA measuring results of a homogeneous and an inhomogeneous media, ethylene glycol and undiluted condensed milk were atomized under identical conditions (4bar, flat cone nozzle Spraying Systems 650050). The velocity and the diameter as well were determined 300 mm below the orifice of the nozzle. The light source was a Laser with a wavelength of 1312 nm. The condensed milk was sprayed at room temperature whereas the ethylene glycol was carried to a temperature of 40°C. At 40°C ethylene glycol has nearly the same liquid properties (viscosity = 13 mPas, surface tension = 45 mN/m, density = 1098 kg/m^3) like condensed milk (15 mPas, 42 mN/m, 1060 kg/m^3) at room temperature. Both liquids behave almost newtonian. So one can consider, that in reality the diameter distribution as well as the velocity distribution of this two fluids has to be identical. The comparison of PDA results demonstrate, that based on the less measuring accuracy in case of the inhomogeneous milk, the diameter distribution is broadened, see Fig. 1.

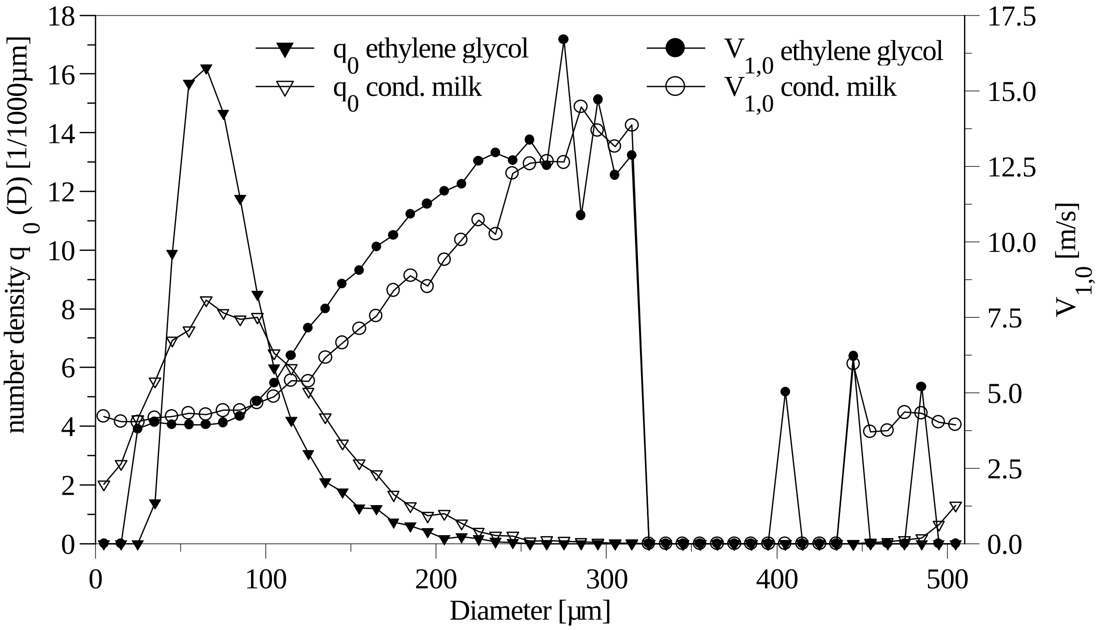

Fig. 1: PDA measured size distribution and Empirical Regression of the mean velocities of an homogeneous media (ethylene glycol at 40°C) and undiluted condensed milk (22% dry substance)

Due to this phenomenon the Empirical Regression of the mean velocities $V_{1,0}(D)$ is flattened out. Because no droplets less than 30 µm are detected no mean velocity can be calculated. Up to a droplet diameter of 260 µm the mean velocity increases continuously with increasing diameter. For droplets larger than 260 µm the velocity and the diameter are not correlated. The reason can be explained by the course of the size distribution. In the range from 260 µm to 510 µm (top of measuring range) only a few droplets are counted. Furthermore, due to random measuring errors diameters are detected not corresponding with the true diameter of the droplet in the measuring volume. So the mean velocity $V_{1,0}$ - calculated from all events in one size bin - often bases on one or two detected droplets. Obviously, these values cannot give statistical statements about the real process.

RECONSTRUCTION OF THE TRUE SIZE DISTRIBUTION

It was already discussed at ILASS '98 that the true size distribution $q_{0,true}$ can be well restored [4]. Under the assumption that the broadening of the size distribution is attributed to a less measuring accuracy in diameter determination the application of a more sophisticated mathematical algorithm enables the PDA user to get information about the correct size distribution. To reconstruct the true size distribution a Fredholm Integral equation of the first kind (convolution) has to be solved.

$$q_{0,\,measured}(D_i) = \sum_{j=1}^{N} TM_{i,\,j} \cdot q_{0,\,true}(D_j) \cdot \Delta D_j \approx \overline{q_{0,\,measured}} = \overline{\overline{(TM)}} \cdot \overline{q_{0,\,true}} \qquad (1)$$

$q_{0,measured}$ is the PDA measured size distribution of the inhomogeneous media; the 'Transfer Matrix' *TM* contains the possibility with which the PDA determines a droplet of diameter D_j as a droplet of diameter D_i; $q_{0,true}$ is the distribution of interest. In our application the values of *TM* are equal to the measuring accuracy in diameter determination of the PDA. A variety of mathematical algorithms exit to solve eqn (1) [1]. In case of PDA measurements the true size distribution can be well restored by application of an iterative algorithm originally proposed by Cahine. The identical procedure was also successfully applied to evaluate measuring results of Fourier Spectroscopy [2].

The distribution of interest $q_{0,true}$ is calculated by the multiple multiplication of every element of the measured size distribution $q_{0,measured}(D_i)$ with a Correction factor C_i.

$$q_{0,\,true}(D_i)^{(n+1)} = C_i^{\,n} \cdot C_i^{(n-1)} \ldots C_i^{\,1} \cdot q_{0,\,measured}(D_i) \qquad ,\,für\ i = 1\ldots N \qquad (2)$$

The values of the Correction factor C_i^n are calculated from the Transfer Matrix TM in consideration of the vector determined by $\overline{\tilde{q}_0} = \overline{\overline{TM}} \cdot \overline{q_{0,\,true}}^{(n)}$ and the measured distribution $q_{0,measured}(D_i)$.

$$C_i^{\,n} = \frac{\sum_{j=1}^{N} TM_{i,j} \cdot r_j}{\sum_{i=1}^{N} TM_{i,j}} \qquad ,\,für\ i = 1\ldots N \qquad (3)$$

$$r_j = \frac{q_{0,\,measured}(D_j)}{\tilde{q}_0(D_j)} \qquad ,\,für\ j = 1\ldots N \qquad (4)$$

The iteration stops if $\left\| \overline{q_{0,\,true}(D_i)^{(n)}} - \overline{q_{0,\,true}(D_i)^{(n-1)}} \right\|$ gets smaller than a definite value.

Fig. 2 shows the size distribution which can be reconstructed from the PDA measured data of condensed milk.

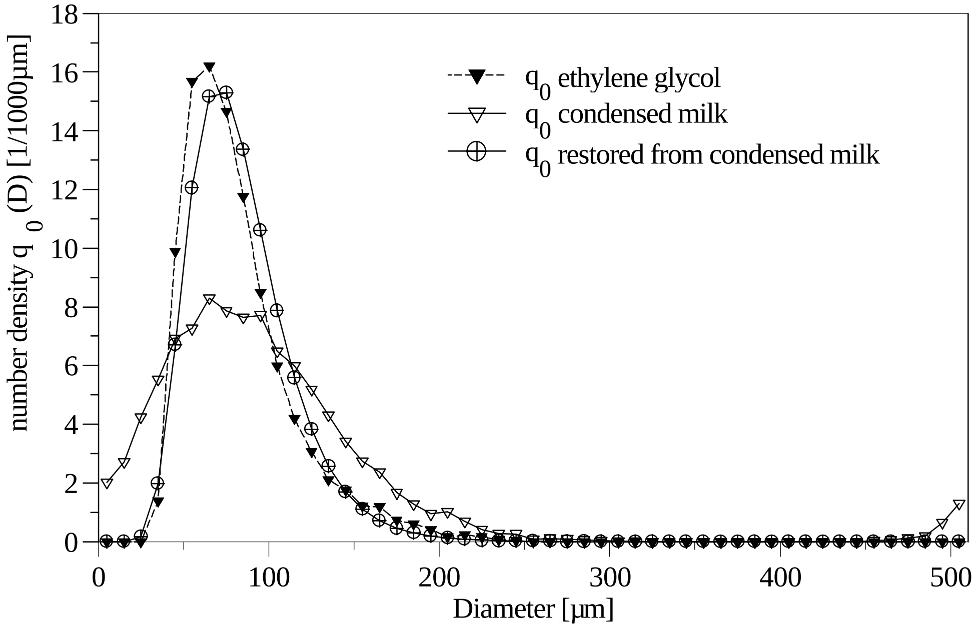

Fig. 2: Measured and reconstructed size distribution $q_{0,true}$ of condensed milk.

VELOCITY-DIAMETER CORRELATION

The velocity of an inhomogeneous droplet can be measured by PDA exactly. Only if the measuring accuracy in diameter determination is worse than 50°-60° phasedifference problems arise in velocity determination. Though the error in diameter determination leads to enormous deviations of the velocity-diameter correlation; the mean velocity calculated from all measured velocities in every size bin differs from reality.

Fig. 3 demonstrates the variations in the velocity-diameter correlation under the assumption that the measuring accuracy decreases from 1° to 30° phase difference. The curve of water is equal to that in Fig. 1. To avoid non statistical values the mean velocity $V_{1,0}$ is only calculated from velocity-size bins with more than 2 measured events. In Fig. 3 is also plotted the result of a mathematical simulation. With this simulation a measuring result was calculated - based on the measured data of water atomization - under the assumption that the measuring accuracy has been decreased from 1° to 30° (equal to 42 µm). Due to the limitation of the PDA measuring range to 360° it seems that droplets with diameters greater than 450 µm exist. Those measuring events are attributed to very small droplets which produce due to the less measuring accuracy phase-differences less than 0. Addition of 360° leads to high phase-differences and greater diameters respectively. The course of the velocity-diameter curve $V_{1,0}(d_i)$ has been influenced too. The presence of droplets smaller than 30 µm with a mean velocity of about 4 m/s is also due to the worse measuring accuracy. For droplets with diameters larger than 90 µm the measured Empirical Regression of the mean velocity is always below that one which was determined for the homogeneous ethylene glycol. Because no events are counted in size bins between 300 µm and 480 µm the mean velocity cannot be calculated. For diameters greater 440 µm the mean velocity stays nearly constant about 4 m/s. This value is comparable with that one of the smallest droplets.

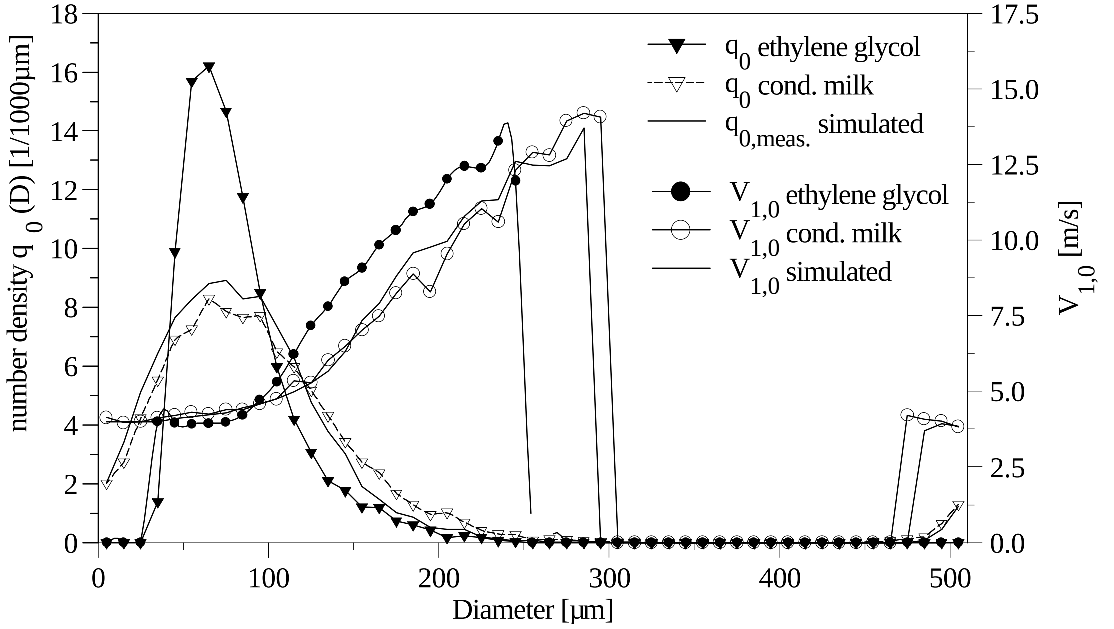

Fig. 3: Simulation of a PDA measuring result based on the data shown in Fig. and the assumption that the measuring accuracy in diameter determination decreases from 1° to 30° (43 µm).

Fig. 3 demonstrates that the result of the simulation is in good agreement with real measurements performed by spraying an inhomogeneous fluid. Experiments with streams of mono dispersed droplets (droplet generator) verified that the diameter of condensed milk droplets can be determined with an accuracy of 30° phasedifference. Fig. 3 shows that both curves - the measured and the simulated one - are in good agreement over the whole measuring range. Also nearly identical values can be observed for diameters greater 480 µm. This fact verifies the validity of the supposed mathematical model.

RECONSTRUCTION OF THE TRUE EMPIRICAL REGRESSION OF THE MEAN VELOCITIES OF EVERY SIZE BIN

The true velocity distribution can be restored from the measured data as well as the real size distribution. The following equations are valid under the assumption that the velocity of an inhomogeneous droplet can be determined without any errors. The inhomogeneities only influence the accuracy in size determination.

It depends on the interest of the PDA user which one of both following mathematical methods should be used.

Calculating the true mean velocities $V_{1,0}(D)$

In most applications the user of an atomizer is only interested in the mean velocity $V_{1,0}$ of every size bin. With Fig. 3 was confirmed that the mean velocity calculated from the measured values can be associated with the true mean velocity by a convolution operation. The mathematical relation is given by eqn (5):

$$\overline{V_{1,0(measured)}} = \overline{\overline{TM}} \cdot \overline{V_{1,0(true),j} \cdot q_{0,true}(D_j) / \left(\sum_j TM_{i,j} \cdot q_{0,true}(D_j)\right)} \qquad (5)$$

During the postprocessing the true size distribution $q_{0,true}(D)$ has to be restored in a first step. Then the only unknown variable is the mean velocity $V_{1,0}$. The transfer matrix TM is equal to that one which has been used to reconstruct the size distribution $q_{0,true}(D)$. It is advantageous to multiply directly the value $q_{0,true}(D_j) / \left(\sum_j TM_{i,j} \cdot q_{0,true}(D_j)\right)$ rowwise with the elements of the transfer matrix, so it can be defined

$$TM_{i,j}{*} = TM_{i,j} \cdot \left(q_{0,true}(D_j) / \left(\sum_j TM_{i,j} \cdot q_{0,true}(D_j)\right)\right) \qquad ,\ für\ i,j = 1, N. \qquad (6)$$

With this simplification eqn (5) can be written as

$$\overline{V_{1,0(measured)}} = \overline{\overline{TM*}} \cdot \overline{V_{1,0(true),i}} \qquad (7)$$

eqn (7) presents again a system of linear equations which can be solved by application of an inversion algorithm. The result of such a reconstruction is given in Fig. 4. The curves based on the data shown in Fig. 1. The mean velocity-diameter correlation of the undiluted condensed milk and ethylene glycol as well was calculated by simultaneous rejection of all size and velocity bins with less than two events.

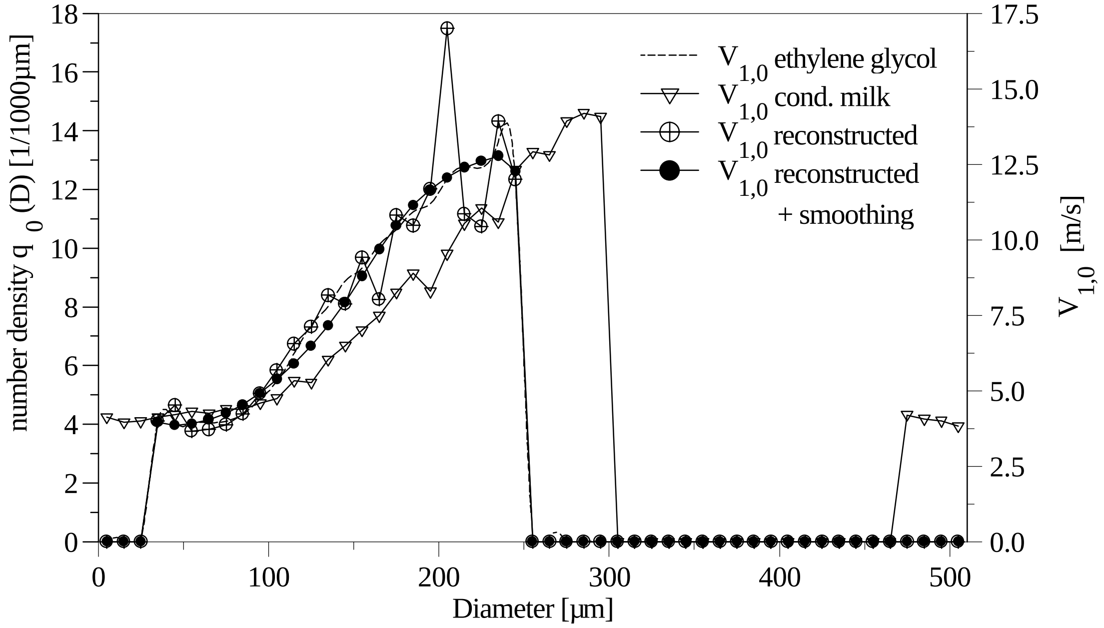

Fig. 4: Reconstruction of the true velocity-diameter correlation from the PDA measured data during the atomization of undiluted condensed milk.

Fig 4 demonstrates, that the true Empirical Regression of the mean velocities can be well restored. To get solutions closer to reality it is recommended to implement a smoothing factor, which gives information about the smoothness of the Empirical Regression, in the reconstruction algorithm.

Calculating the velocity distribution V_j of every size bin D_i

It is also possible to restore the whole true velocity-diameter correlation (-matrix). With this information the PDA user gets information about the velocity distribution for droplets of definite diameter $V_j(D_i)$. Under the assumption, that the measuring accuracy in velocity determination is not influenced by the optical properties of the liquid, eqn (8) describes the relation between the measured velocity-size matrix of an inhomogeneous liquid and the true velocity-size matrix which is expected to be equal to that one of a homogeneous media with identical physical properties.

$$\overline{\overline{DV_{measured}}} = \overline{\overline{DV_{true}}} \cdot \overline{\overline{TM}} \tag{8}$$

The values $DV_{i,j}$ describe the number of events in the velocity bin i and the size bin j. For a definite velocity bin i eqn (8) can be simplified to eqn (9)

$$\overline{q_{0,\,measured}(D)_i} = \overline{\overline{TM}} \cdot \overline{q_{0,\,real}(D)_i} \tag{9}$$

respectively.

$$q_{0,\,measured}(D_k)_i = \sum_{j=1}^{j=N} (TM_{k,j})_i \cdot q_{0,\,true}(D_j)_i \qquad ,\, für\ i,k = 1\ldots N \tag{10}$$

The measured number density $q_{0,measured}$ of droplets with diameter D_k and the velocity V_i is the sum of the product of the true number density $q_{0,true}(D_j)$ and the probability that a droplet of diameter D_j and velocity V_i is detected incorrectly as a droplet of diameter D_k. To reconstruct the true number density of the velocities in every size bin, one has to perform an inversion algorithm to the diameter distribution for constant velocity.

Afterwards, it is possible to determine in a further step the true Empirical Regression of the mean velocities. This true Empirical Regression can be calculated by determination of the mean velocity of all droplets in one size bin.

SUMMARY AND CONCLUSIONS

Measuring droplets of inhomogeneous media by PDA the measuring accuracy in diameter determination decreases with increasing concentration of the inhomogeneities. As a consequence the boarders of size distributions are flattened out. Furthermore those bias influences in the same manner the velocity-diameter correlation, which is approximated by an ’Empirical Regression’ of the mean velocities of every size bin $V_{1,0}(D_i)$. In most cases the mean velocities $V_{1,0(measured)}(D_i)$ calculated from the measured data are much smaller than the mean velocities $V_{1,0(true)}(D_i)$ truely occuring during the atomization process.

The differences between the measured size distribution or the measured Empirical Regression and the true size distribution or the true Empirical Regression can be explained by a simple mathematical model: a convolution. The solution of the mathematical equation can be performed by the application of more sophisticated inversion algorithms.

The comparison of the measured size distribution and the Empirical Regression of ethylene glycol and those reconstructed from the data of undiluted condensed milk demonstrate that PDA in combination with post-processing algorithms can be applied to investigate the atomization of inhomogeneous media.

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