The application of the modified discrete sources method to the problem of wave diffraction on a body in chiral half-space.

Alexander G. Kyurkchan [a,]* and Sergey A. Manenkov[a]
[a]*Moscow Technical University of Communication and Informatics, 8A, Aviamotornaya Street, Moscow, 111024, Russia*

Abstract

The new modification of the discrete sources method has been applied to solve the problem of wave diffraction on a cylindrical body located in the homogeneous chiral half-space. The method is compared with the technique, based on generalized Rayleigh series for the case of the cylinder with round cross-section. The numerical results show that the proposed method provides a high accuracy of computation.

Keywords: Scattering problems; Chiral media; Discrete sources method; Analytical continuation of scattered field

1. Introduction

The modified discrete sources method (MDSM) has been originally proposed in [1] and then applied to various diffraction problems, for example to the diffraction on a single body and periodical surface [1-3]. The fundamental feature of the method consists in construction of the auxiliary contour – the carrier of the discrete sources. The choice of the auxiliary contour is accomplished by the uniform procedure residing in analytical deformation of the scatterer boundary. The results of the investigations which are made in [1-3] show the high efficiency of MDSM. For instance the computational algorithms based on MDSM retain stability in the whole range of the problem parameters. Besides the method allows to perform calculations with a very high accuracy. In this work MDSM is extended to the problem of wave diffraction by the infinite cylindrical obstacle located in a homogeneous chiral half-space. Note that the problem of wave

* Corresponding author. Tel.: +95-236-2267.
E-mail address: kurkchan@mtu-net.ru (A.G.Kyurkchan)

scattering in the chiral media is a vector one. However, taking into consideration that the model in hand is two-dimensional, one can formulate the problem in terms of four scalar potentials, which are connected both at the interface of the chiral half-space and at the surface of the cylindrical body.

2. Formulation of the problem

We consider the field produced by arbitrary sources located in the half-space $y > d$ and diffracted from a cylindrical scatterer embedded in the half-space $y < d$ (Fig.1). The contour of the cylinder cross section is denoted by S. Let the origin be inside S and the z axis be aligned with the cylinder generatrix. The interface between the media is the plane $y = d$. It is assumed that a chiral medium fills the lower half-space and is characterized by permittivity ε_2, permeability μ_2 and chirality parameter κ [4]. The half-space $y > d$ is filled with an isotropic magnetodielectric with parameters ε_1 and μ_1. Electromagnetic fields $\vec{E}$ and $\vec{H}$ satisfy the usual Maxwell equations in the half-space $y > d$ and the generalized Maxwell equations for a chiral medium at $y < d$. The latter have the following form [5]:

$$\begin{aligned} \operatorname{curl}\vec{H} &= ik\varepsilon_2\vec{E} + k\kappa\vec{H}, \\ \operatorname{curl}\vec{E} &= -ik\mu_2\vec{H} + k\kappa\vec{E}, \end{aligned} \tag{1}$$

where k is the wave number in free space. We assume that the tangential components of the electric and magnetic fields satisfy the continuity conditions at the medium interface and the tangential component of the electric field vanishes on the cylinder surface; i.e.,

$$(\vec{n} \times \vec{E})\Big|_S = 0 \tag{2}$$

where $\vec{n}$ is the outward normal. Since we analyze a two-dimensional (2D) model, it suffices to find E_z and H_z components of the electromagnetic field. The remaining components can be expressed through E_z and H_z. Let us perform the following substitution:

$$E_z = u_+ + u_-;$$
$$H_z = i\eta_2^{-1}(u_+ - u_-). \tag{3}$$

where $\eta_2 = \sqrt{\mu_2 / \varepsilon_2}$. In (3), it is assumed that $y < d$ i.e., the lower half-space is considered. One can show that functions u_+ and u_- satisfy two independent Helmholtz equations:

$$\Delta u_+ + k_+^2 u_+ = 0,$$
$$\Delta u_- + k_-^2 u_- = 0, \tag{4}$$

where $k_\pm = k(\nu_2 \pm \kappa)$, $\nu_2 = \sqrt{\mu_2 \varepsilon_2}$, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Components E_x and H_x in the chiral half-space can be represented as follows:

$$E_x = \frac{1}{k_+}\frac{\partial u_+}{\partial y} - \frac{1}{k_-}\frac{\partial u_-}{\partial y},$$
$$H_x = \frac{1}{\eta_2}\left(\frac{1}{k_+}\frac{\partial u_+}{\partial y} + \frac{1}{k_-}\frac{\partial u_-}{\partial y}\right). \tag{5}$$

Similarly we denote (in the region $y > d$)

$$E_z = u_E ,$$
$$H_z = i\eta_1^{-1} u_H , \tag{6}$$

where E_z and H_z are the corresponding components of vectors $\vec{E}$ and $\vec{H}$ in the upper (dielectric) half-space and $\eta_1 = \sqrt{\mu_1 / \varepsilon_1}$. Functions u_E and u_H satisfy the equation

$$\Delta u_{E,H} + k_1^2 u_{E,H} = 0 \tag{7}$$

where $k_1 = k\sqrt{\varepsilon_1 \mu_1}$. Using formulas (3), (5), and (6) and the Maxwell equations, one can show that, for potentials u_+, u_-, u_E and u_H, the following boundary conditions on the plane $y = d$ are fulfilled:

$$
\begin{aligned}
&u_E = u_+ + u_- , \\
&\eta_1^{-1} u_H = \eta_2^{-1}(u_+ - u_-), \\
&\frac{1}{k_1}\frac{\partial u_H}{\partial y} = \frac{1}{k_+}\frac{\partial u_+}{\partial y} - \frac{1}{k_-}\frac{\partial u_-}{\partial y}, \\
&\frac{1}{\eta_1 k_1}\frac{\partial u_E}{\partial y} = \frac{1}{\eta_2}\left(\frac{1}{k_+}\frac{\partial u_+}{\partial y} + \frac{1}{k_-}\frac{\partial u_-}{\partial y}\right).
\end{aligned}
\tag{8}
$$

Thus, the original diffraction problem is reduced to determination of four wave potentials satisfying equations (4) and (7), boundary conditions (8), and the radiation condition at infinity.

3. Basic computational relationships

We shall solve the stated problem by the MDSM. This method is one of the realization of the auxiliary currents method (ACM). ACM reduces the initial boundary problem to the integral equation for some unknown function (the current). If a scalar 2D problem is considered the following theorem is true [1-3].

Let a simple closed curve Σ is such that the wave number k is not the eigenvalue of the internal homogeneous Dirichlet problem for the domain inside Σ. Then the integral equation for the unknown auxiliary current is solvable if and only if the contour Σ encloses all the singularities of the solution of the corresponding boundary problem.

It is obvious that the conditions of the theorem remain necessary for the stability of the algebraic systems of the discrete source method when the number of the sources is increasing. In accordance with MDSM the discrete sources are placed at the auxiliary contour S_0, which parametrically represented as [1-3]:

$$
\begin{aligned}
x_в(\varphi) &= \mathrm{Re}[\rho(\varphi + i\delta)\exp(-\delta + i\varphi)], \\
y_в(\varphi) &= \mathrm{Im}[\rho(\varphi + i\delta)\exp(-\delta + i\varphi)],
\end{aligned}
\tag{9}
$$

where $r = \rho(\varphi)$ is the equation of the body contour in the polar coordinates, δ is a positive parameter such that the auxiliary contour encloses the singularities of the secondary diffraction field, and $\varphi \in (0, 2\pi)$. When a body is located in a homogeneous isotropic medium, the scattered field can be presented in the form of the integral

$$u^1(\vec{r}) = \int_{S_0} G(\vec{r}, \vec{r}') j(\vec{r}') ds', \tag{10}$$

where $G = H_0^{(2)}(k|\vec{r} - \vec{r}'|)$ is the Green's function of a homogeneous dielectric space and $j(\vec{r}')$ is an unknown current density. Emphasize that the contour S_0 in (10) is assumed to be nonresonant in accordance with the theorem. The substitution of the Riemann sum for the integral in (10) reduces the original boundary problem to an algebraic system of the first kind for the unknown amplitudes of discrete sources. Evidently, this technique can be extended to the problem of scattering from a body embedded in a chiral medium. Using Lorentz lemma and equations (1) as well as boundary conditions (2), (8), one can obtain the following representations for wave potentials u_+ and u_- in a chiral half-space:

$$\begin{aligned} u_+(\vec{r}) = u_+^0(\vec{r}) - \frac{\eta_2}{8} \int_S \Big\{ & j_z^e(\vec{r}')\big(k_+ G_+(\vec{r}, \vec{r}') + k_- G_\pm(\vec{r}, \vec{r}')\big) + \\ & + j_\varphi^e(\vec{r}') \frac{\partial}{\partial n'}\big(G_+(\vec{r}, \vec{r}') - G_\pm(\vec{r}, \vec{r}')\big) \Big\} ds', \end{aligned} \tag{11}$$

$$\begin{aligned} u_-(\vec{r}) = u_-^0(\vec{r}) - \frac{\eta_2}{8} \int_S \Big\{ & j_z^e(\vec{r}')\big(k_- G_-(\vec{r}, \vec{r}') + k_+ G_\mp(\vec{r}, \vec{r}')\big) - \\ & - j_\varphi^e(\vec{r}') \frac{\partial}{\partial n'}\big(G_-(\vec{r}, \vec{r}') - G_\mp(\vec{r}, \vec{r}')\big) \Big\} ds'. \end{aligned} \tag{12}$$

where $j_z^e = (\vec{n} \times \vec{H})_z\big|_S$ and $j_\varphi^e = H_z\big|_S$ ($\vec{j}^{\,e}$ is the electric current which is localized on S and induce the scattered electromagnetic fields $\vec{E}^1$ and $\vec{H}^1$). The functions u_+^0 and u_-^0 in formulas (11), (12) are the primary wave fields in the lower medium determined in the absence of the scatterer. Functions G_+, $G_\mp$, G_- and $G_\pm$ are the components of the matrix Green's function

$$\hat{G} = \begin{pmatrix} G_{+} & G_{\pm} \\ G_{\mp} & G_{-} \end{pmatrix},$$

satisfying the following wave equations in the chiral half-space $y < d$:

$$\Delta G_{+} + k_{+}^{2} G_{+} = -4i\delta(\vec{r} - \vec{r}'),$$
$$\Delta G_{\mp} + k_{-}^{2} G_{\mp} = 0, \tag{13}$$

and

$$\Delta G_{\pm} + k_{+}^{2} G_{\pm} = 0,$$
$$\Delta G_{-} + k_{-}^{2} G_{-} = -4i\delta(\vec{r} - \vec{r}'). \tag{14}$$

In addition, the pairs of the above components (G_{+} and $G_{\mp}$, G_{-} and $G_{\pm}$), as well as the corresponding solutions of the homogeneous Helmholtz equation (7), satisfy boundary condition (8) at $y = d$. Thus, for G_{+} and $G_{\pm}$ we have

$$G_{+}(\vec{r}, \vec{r}') = H_{0}^{(2)}(k_{+}|\vec{r} - \vec{r}'|) + \frac{1}{\pi}\int_{-\infty}^{\infty} R_{+}(\alpha)\exp\left(i(y + y')\sqrt{k_{+}^{2} - \alpha^{2}}\right)\times$$
$$\times\exp\left(-i\alpha(x - x')\right)\frac{d\alpha}{\sqrt{k_{+}^{2} - \alpha^{2}}}, \tag{15}$$

$$G_{\pm}(\vec{r}, \vec{r}') = \frac{1}{\pi}\int_{-\infty}^{\infty} R_{\pm}(\alpha)\exp\left(iy\sqrt{k_{+}^{2} - \alpha^{2}} + iy'\sqrt{k_{-}^{2} - \alpha^{2}}\right)\times$$
$$\times\exp\left(-i\alpha(x - x')\right)\frac{d\alpha}{\sqrt{k_{-}^{2} - \alpha^{2}}}. \tag{16}$$

where $y < d$. The signs of the square roots are chosen so as to provide for their nonpositive imaginary parts. Functions R_{+} and $R_{\pm}$. are the reflection and transmission factors, respectively, for a partial plane wave incident from the chiral half-space on the interface between the media. They are determined by the formulas

$$R_{+} = A_{+}\exp(-i2d\sqrt{k_{+}^{2}-\alpha^{2}}),$$
$$R_{\pm} = A_{\pm}\exp(-id(\sqrt{k_{+}^{2}-\alpha^{2}}+\sqrt{k_{-}^{2}-\alpha^{2}})), \tag{17}$$

where

$$A_{+} = \frac{2\lambda_{+}\lambda_{-}-2\lambda_{1}^{2}+\lambda_{1}(\lambda_{+}-\lambda_{-})(\eta_{21}+\eta_{21}^{-1})}{2\lambda_{+}\lambda_{-}+2\lambda_{1}^{2}+\lambda_{1}(\lambda_{+}+\lambda_{-})(\eta_{21}+\eta_{21}^{-1})},$$
$$A_{\pm} = \frac{2\lambda_{1}\lambda_{-}(\eta_{21}^{-1}-\eta_{21})}{2\lambda_{+}\lambda_{-}+2\lambda_{1}^{2}+\lambda_{1}(\lambda_{+}+\lambda_{-})(\eta_{21}+\eta_{21}^{-1})}, \tag{18}$$

$$\lambda_{\pm} = \sqrt{k_{\pm}^{2}-\alpha^{2}}/k_{\pm},\ \ \lambda_{1} = \sqrt{k_{1}^{2}-\alpha^{2}}/k_{1},\ \ \eta_{21} = \eta_{2}/\eta_{1}.$$

The remaining components of the matrix Green's function can be obtained from (15) and (16) by reversing the subscript signs. Note that the off-diagonal elements of the Green's function are connected by the formula:

$$k_{-}G_{\pm}(\vec{r},\vec{r}\,') = k_{+}G_{\mp}(\vec{r}\,',\vec{r}), \tag{19}$$

which follows from the principle of reciprocity for chiral medium.

In order to simplify the further analysis, let us introduce the vectors

$$\vec{u} = \begin{bmatrix} u_{+} \\ u_{-} \end{bmatrix}, \qquad \vec{u}^{\,0} = \begin{bmatrix} u_{+}^{0} \\ u_{-}^{0} \end{bmatrix}, \qquad \vec{j} = \begin{bmatrix} j_{+} \\ j_{-} \end{bmatrix}, \tag{20}$$

where j_{+} and j_{-} are the unknown current densities on the auxiliary contour. After some algebra, we arrive at the following representations of the fields in the chiral medium:

$$\vec{u}(\vec{r}) = \vec{u}^{\,0}(\vec{r}) + \int_{S_0}\hat{G}(\vec{r},\vec{r}\,')\vec{j}(\vec{r}\,')ds' \tag{21}$$

where the line of integration is the auxiliary contour described above. Functions u_{+} and u_{-} involved in (21) satisfy the homogeneous wave equations (4) outside the auxiliary contour and the boundary conditions (8) at $y = d$. Now, let us reduce the problem to solution of an algebraic system of equations. To this end, we replace the integral in (21) by the Riemann sum and substitute the result in the conditions imposed on the cylinder surface, which have the form [4]

$$\begin{aligned} &(u_+ + u_-)\Big|_S = 0, \\ &\left(\frac{1}{k_+}\frac{\partial u_+}{\partial n} - \frac{1}{k_-}\frac{\partial u_-}{\partial n}\right)\Bigg|_S = 0, \end{aligned} \tag{22}$$

where $\partial / \partial n$ denotes differentiation with respect to the outward normal. Satisfying the obtained equalities at collocation points on the original contour (which are spaced at equal angular intervals) [1], we arrive at the following system:

$$\begin{aligned} &\sum_{n=1}^{N} u_{mn}^{+} j_n^{+} + \sum_{n=1}^{N} u_{mn}^{-} j_n^{-} = -a_m^0, \\ &\sum_{n=1}^{N} v_{mn}^{+} j_n^{+} + \sum_{n=1}^{N} v_{mn}^{-} j_n^{-} = -b_m^0, \end{aligned} \qquad m = 1,2...,N. \tag{23}$$

where

$$\begin{aligned} u_{mn}^{+} &= G_{+}(\vec{r}_m, \vec{r}_n) + G_{\mp}(\vec{r}_m, \vec{r}_n), \\ u_{mn}^{-} &= G_{-}(\vec{r}_m, \vec{r}_n) + G_{\pm}(\vec{r}_m, \vec{r}_n), \end{aligned} \tag{24}$$

$$\begin{aligned} v_{mn}^{+} &= \frac{1}{k_+}\hat{D}_m G_{+}(\vec{r}, \vec{r}_n) - \frac{1}{k_-}\hat{D}_m G_{\mp}(\vec{r}, \vec{r}_n), \\ v_{mn}^{-} &= \frac{1}{k_+}\hat{D}_m G_{\pm}(\vec{r}, \vec{r}_n) - \frac{1}{k_-}\hat{D}_m G_{-}(\vec{r}, \vec{r}_n), \end{aligned} \tag{25}$$

$$\hat{D}_m = \left(\rho(\varphi_m)\frac{\partial}{\partial r} - \frac{\rho'(\varphi_m)}{\rho(\varphi_m)}\frac{\partial}{\partial \varphi}\right)_{\vec{r}_m}.$$

The right-hand side of the system is expressed by

$$\begin{aligned} a_m^0 &= u_+^0(\vec{r}_m) + u_-^0(\vec{r}_m), \\ b_m^0 &= \frac{1}{k_+}\hat{D}_m u_+^0(\vec{r}) - \frac{1}{k_-}\hat{D}_m u_-^0(\vec{r}), \end{aligned} \tag{26}$$

In (24) - (26), $\vec{r}_m$ and $\vec{r}_n$ denote the radius vectors of collocation points and coordinates of discrete sources, respectively. The latter are located on auxiliary contour S_0. In (23) N is the number of discrete sources.

The scattered fields in the upper (dielectric) half-space are usually of interest. They can be written as follows:

$$u_E^1(\vec{r}) = \sum_{n=1}^{N} j_n^+ G_E^+(\vec{r},\vec{r}_n) + \sum_{n=1}^{N} j_n^- G_E^-(\vec{r},\vec{r}_n),$$
$$u_H^1(\vec{r}) = \sum_{n=1}^{N} j_n^+ G_H^+(\vec{r},\vec{r}_n) + \sum_{n=1}^{N} j_n^- G_H^-(\vec{r},\vec{r}_n). \tag{27}$$

Here G_E^+, G_E^-, G_H^+ and G_H^- are the components of the matrix Green's function at $y > d$. We present the expressions for functions G_E^+, G_H^+, since the remaining components are determined by similar relations.

$$G_E^+(\vec{r},\vec{r}') = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(1 + A_+ + A_\mp\right) \exp\left(-i(y-d)\sqrt{k_1^2-\alpha^2} + i(y'-d)\sqrt{k_+^2-\alpha^2}\right) \times$$
$$\times \exp\left(-i\alpha(x-x')\right) \frac{d\alpha}{\sqrt{k_+^2-\alpha^2}}, \tag{28}$$

$$G_H^+(\vec{r},\vec{r}') = \frac{1}{\eta_{21}\pi} \int_{-\infty}^{\infty} \left(1 + A_+ - A_\mp\right) \exp\left(-i(y-d)\sqrt{k_1^2-\alpha^2} + i(y'-d)\sqrt{k_+^2-\alpha^2}\right) \times$$
$$\times \exp\left(-i\alpha(x-x')\right) \frac{d\alpha}{\sqrt{k_+^2-\alpha^2}}, \tag{29}$$

Thus the algebraic system (23) and the formulas (27) - (29) allow to solve the diffraction problem.

4. The solution of the problem for the body with circular cross-section and comparison of the methods

To test the method we study diffraction on the scatterer with circular cross section. Let the radius of the cylinder is r_0. The coordinate system is the same as in the previous case (the origin is in the center of the cross-section of the cylinder). The solution for this geometry can be obtained by the following technique. One can present the components G_+ and $G_\pm$ of the matrix Green's function in the form:

$$G_+(\vec{r},\vec{r}\,') = \sum_{n=-\infty}^{\infty} i^n J_n(k_+ r')\exp(-in\varphi')\cdot u_n^+(r,\varphi)\ ,$$
$$G_\pm(\vec{r},\vec{r}\,') = \sum_{n=-\infty}^{\infty} i^n J_n(k_- r')\exp(-in\varphi')\cdot u_n^\pm(r,\varphi)\ , \qquad (30)$$

where

$$u_n^+(r,\varphi) = i^{-n} H_n^{(2)}(k_+ r)\exp(in\varphi) + \frac{1}{\pi}.\int_{-i\infty}^{\pi+i\infty} R_+(\psi)\exp(-ik_+ r\cos(\psi+\varphi)+in\psi)d\psi,$$
$$u_n^\pm(r,\varphi) = \frac{1}{\pi}.\int_{-i\infty}^{\pi+i\infty} R_\pm(\psi)\exp(-ik_+ r\cos(\psi_+ +\varphi)+in\psi)d\psi\ , \qquad (31)$$

and

$$k_+\cos\psi_+ = k_-\cos\psi, \qquad k_+\sin\psi_+ = \sqrt{k_+^2 - k_-^2\cos^2\psi} \qquad (32)$$

In (31) we denote $R_+(\psi) \equiv R_+(k_+\cos\psi)$ and $R_\pm(\psi) \equiv R_\pm(k_-\cos\psi)$. The expressions for the components $G_\mp$ and G_- can be obtained by the reversing the subscript signs. Then we substitute these relations in the formulas (11) and (12). We get:

$$u_+(r,\varphi) = u_+^0(r,\varphi) + \sum_{n=-\infty}^{\infty} a_n^+ u_n^+(r,\varphi) + \sum_{n=-\infty}^{\infty} a_n^- u_n^\pm(r,\varphi), \qquad (33)$$

$$u_-(r,\varphi) = u_-^0(r,\varphi) + \sum_{n=-\infty}^{\infty} a_n^- u_n^-(r,\varphi) + \sum_{n=-\infty}^{\infty} a_n^+ u_n^\mp(r,\varphi), \qquad (34)$$

where the values a_n^+ and a_n^- are:

$$a_n^+ = -\frac{k_+ r_0\eta_2}{8} i^n \int_0^{2\pi} \Big(J_n(k_+ r_0) j_z^e(r_0,\varphi) + J_n'(k_+ r_0) j_\varphi^e(r_0,\varphi)\Big)\exp(-in\varphi)d\varphi,$$
$$a_n^- = -\frac{k_- r_0\eta_2}{8} i^n \int_0^{2\pi} \Big(J_n(k_- r_0) j_z^e(r_0,\varphi) - J_n'(k_- r_0) j_\varphi^e(r_0,\varphi)\Big)\exp(-in\varphi)d\varphi. \qquad (35)$$

Thus the scattered fields in the chiral half-space are presented by the formulas, which generalized the usual Rayleigh series for diffraction in homogeneous dielectric medium. These series converge absolutely and uniformly in the half-space $y < d$ with $r \geq r_0$. Now we get the algebraic system for the unknown coefficients a_n^+ and a_n^-. To this end we substitute the expressions (33) and (34) in the conditions (22) imposed on the scatterer surface. After some algebra we get:

$$\begin{aligned} &\sum_{n=-\infty}^{\infty} A_{mn}^{+} a_n^{+} + \sum_{n=-\infty}^{\infty} A_{mn}^{-} a_n^{-} = -p_m, \\ &\sum_{n=-\infty}^{\infty} B_{mn}^{+} a_n^{+} + \sum_{n=-\infty}^{\infty} B_{mn}^{-} a_n^{-} = -q_m, \end{aligned} \qquad m = 0, \pm 1, \pm 2.... \tag{36}$$

where

$$\begin{aligned} A_{mn}^{+} &= H_m^{(2)}(k_+ r_0)\delta_{mn} + J_m(k_+ r_0) W_{mn}^{+} + J_m(k_- r_0) W_{mn}^{\mp}, \\ A_{mn}^{-} &= H_m^{(2)}(k_- r_0)\delta_{mn} + J_m(k_- r_0) W_{mn}^{-} + J_m(k_+ r_0) W_{mn}^{\pm}, \\ B_{mn}^{+} &= H_m^{(2)\prime}(k_+ r_0)\delta_{mn} + J_m'(k_+ r_0) W_{mn}^{+} - J_m'(k_- r_0) W_{mn}^{\mp}, \\ B_{mn}^{-} &= -H_m^{(2)\prime}(k_- r_0)\delta_{mn} - J_m'(k_- r_0) W_{mn}^{-} + J_m'(k_+ r_0) W_{mn}^{\pm}, \end{aligned} \tag{37}$$

$$\begin{aligned} W_{mn}^{+} &= \frac{1}{\pi} \int_{-i\infty}^{\pi+i\infty} R_+(\psi) \exp(i(m+n)\psi) d\psi, \\ W_{mn}^{-} &= \frac{1}{\pi} \int_{-i\infty}^{\pi+i\infty} R_-(\psi) \exp(i(m+n)\psi) d\psi, \\ W_{mn}^{\pm} &= \frac{1}{\pi} \int_{-i\infty}^{\pi+i\infty} R_{\pm}(\psi) \left(\tau_{\pm}^{-1}\cos\psi + i\sqrt{1-\tau_{\pm}^{-2}\cos^2\psi}\right)^m \exp(in\psi) d\psi, \\ W_{mn}^{\mp} &= \frac{1}{\pi} \int_{-i\infty}^{\pi+i\infty} R_{\mp}(\psi) \left(\tau_{\pm}\cos\psi + i\sqrt{1-\tau_{\pm}^{2}\cos^2\psi}\right)^m \exp(in\psi) d\psi, \end{aligned} \tag{38}$$

In these formulas $\tau_{\pm} = k_+ / k_-$ and δ_{mn} is the Kronecker delta. The values p_m and q_m are the Fourier coefficients:

$$p_m = \frac{1}{2\pi} \int_0^{2\pi} \left(u_+^0 + u_-^0\right)_{r=r_0} \exp(-im(\varphi - \pi/2)) d\varphi,$$

$$q_m = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{1}{k_+} \frac{\partial u_+^0}{\partial r} - \frac{1}{k_-} \frac{\partial u_-^0}{\partial r} \right)_{r=r_0} \exp(-im(\varphi - \pi/2)) d\varphi. \qquad (39)$$

It can be shown that the infinite system (36) is solvable by the reduction method.

We compared the above methods for the problem of scattering of the plane wave by circular metallic cylinder. The incident field is presented as:

$$E_z^{\text{inc.}} = \exp(-ik_1(x \sin\theta - y \cos\theta)), \quad H_z^{\text{inc.}} = 0. \qquad (40)$$

which means that an *E*-polarized wave is incident on the structure. We examined the value of the intensities of the *E* and *H* polarized components of the diffracted electromagnetic field in the upper (dielectric) space. We denote them I_E and I_H respectively. In far zone the wave potentials are represented as

$$u_{E,H}^1 \sim F_{E,H}(\varphi) \cdot \sqrt{\frac{2}{\pi k_1 r}} \cdot \exp(-ik_1 r + i\pi/4), \qquad (41)$$

where the patterns $F_{E,H}(\varphi)$ can be found by passage to the asymptotic in the presentations for scattered fields in the upper half-space. Calculation results were obtained for the following problem parameters: the radius of the cylinder is $r_0 = 2$, the distance to the interface is $d = 3$, and the wave incidence angle is $\theta = 45^\circ$. The calculation results are summarized in the table, where the data provided by the method of generalized Rayleigh series (MGRS) are also presented. One can see that both of the methods yield virtually coincident results.

5. Numerical results

The method developed is applied to simulate diffraction from infinite cylinders with cross section contours in the form of an ellipse and a multi-lobe curve. The latter is described by the equation $\rho(\varphi) = a(1 + \tau \cos q\varphi)$. We took $q = 2$ and $q = 3$. The problem is characterized by the following parameters: the wavelength $\lambda = 6.28319$ mm, medium permittivities $\varepsilon_1 = \varepsilon_2 = 1$, permeabilities $\mu_1 = 1$ and $\mu_2 = 5$. The MDSM ensures calculations at a high accuracy when the number of basis functions is rather small. This circumstance is illustrated in Fig. 2, where the residual $\sigma_{.}$ of the wave field on the body contour is shown. The residual is estimated at the points $(\rho(\varphi_{n-1/2}), \varphi_{n-1/2})$, where $\varphi_n = 2\pi n / N$ $(n = 0,1,..., N)$ are the angular coordinates of the collocation points. Scattering from the cylinder with a double-lobe cross section is analyzed ($q = 2$). The parameter $a = 1$ mm and $\tau = 0,5$. *E*-polarized plane wave is incident at an angle of 45°. The distance $d = 2$ mm and the chirality parameter is $\kappa = 1$. The curves 1 and 2 correspond to the first and second condition in the formula (22). As is seen in the figure, the residual maximum does not exceed 10^{-5}. The number of discrete sources is relatively small (45). Note that the parameter τ of the contour of the body is chosen so that its border do not satisfies Rayleigh hypothesis.

Let us analyze the diffraction characteristics of the wave field depending on various parameters of the problem. Fig.3 demonstrates the angular dependences of the *E*- and *H*-polarized components of the diffracted field (the curves 1 and 2 in the figure) in the upper half-space. It is considered the scattering of *E*-polarized plane wave (40) on the cylinder with the double-lobe cross-section. The curves are plotted for various values of the incident angle θ. The distance between the medium interface and the point $(0, \rho(\pi/2))$ on the body contour is chosen $\Delta = \lambda_{\_} / 4$ where $\lambda_{\_} = 2\pi / k_{\_}$. The sizes of the body and the chirality parameter have the same values as in the previous case. Fig.4 shows the dependences of the above components of the diffracted field versus chirality parameter κ. The plane wave is incident at the angle $\theta = 30^{\circ}$. The curves 1 and 2 correspond to the dependence of the *E*- and *H*-polarized components respectively. As is seen in

Fig.3 and Fig.4 the level of the depolarized component (I_H) is lower than that of the *E*-polarized component approximately by two orders of magnitude. One can also see that the dependences for the *E*-polarized component are much less “sensitive” to variations of the chirality parameter.

Fig.5*a* and 5*b* demonstrates the influence of the shape of the cross-section of the body on the scattered field. The contour S is the multi-lobe with $q = 3$. The parameter $a = 1$ mm and τ changes from 0.3 to 0.7. The other parameters are the following: $d = 2$ mm, $\theta = 45^\circ$ and $\kappa = 1$. The dashed curves in the figures correspond to the scattering on the circular cylinder with the radius $r_0 = 1.5$.

It is of interest to investigate scattering of circularly polarized electromagnetic waves in a chiral medium in the presence of the depolarization effect. To this end, the angular dependences of diffraction field intensities are obtained in the case when a circularly polarized plane wave is incident on an elliptic cylinder located in a chiral half-space. The corresponding curves are depicted in Figs. 6, 7. The Fig. 6 is plotted for the case when $\Delta = \lambda_+ / 4$ ($\lambda_+ = 2\pi / k_+$) and Fig. 7 conforms to the case when $\Delta = \lambda_- / 4$. The semi-axes of the ellipse are $a = 1.5$ mm and $b = 0.5$ mm. The incident field is represented in the form

$$E_z^{\text{inc.}} = \exp(-ik_1(x\sin\theta - y\cos\theta)),$$
$$H_z^{\text{inc.}} = \pm i\eta_1^{-1}\exp(-ik_1(x\sin\theta - y\cos\theta)). \qquad (42)$$

Figures 6*a* and 7*a* (6*b* and 7*b*) refer to the situation when the plus (minus) sign is used in (42). The curves are plotted for the horizontal (dotted lines) and vertical (solid lines) positions of the elliptic cylinder. In the former and latter cases, the major and minor axes of the ellipse respectively is parallel to the interface. The curves 1 in the figures correspond to the dependence $I_E(\varphi)$ and the curves 2 conform to $I_H(\varphi)$. The scattered field intensities observed at the horizontal position of the cylinder exceed those observed at its vertical position by an order of magnitude. One can also see that the direction of the electric field vector rotation substantially affects the field scattered by the cylinder.

6. Conclusions

We have studied wave scattering from an infinite cylindrical body immersed in a homogeneous chiral half-space. The representations for the scattered field in a chiral medium are obtained. Using these relations we have expressed the scattered fields in terms of the fields produced by discrete sources located on an auxiliary contour. The algebraic system for unknown amplitudes of the discrete sources is obtained. In the special case when the cross-section of the body is circular the problem has solved by the use of the method of generalized Rayleigh series. Comparison of the methods shows the high accuracy of the calculations results obtained by MDSM. The efficiency of the method developed is demonstrated for wave diffraction from cylinders with elliptic and multi-lobe cross sections. The proposed approach can be generalized to the case of a multi-layer chiral media and three-dimensional problems.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research, project no 03-02-16336, 03-02-06142.

References.

[1] Kyurkchan AG, Minaev SA, Soloveitchik AL. A modification of the method of discrete sources based on a priory information about the singularities of the diffracted field. Journ of Commun Techn and Electron 2001; 46: 615-21.

[2] Anioutine AP, Kyurkchan AG, Minaev SA. A modified method of discrete sources. Journ of Commun. Techn and Electron 2002; 47: 864-69.

[3] Anioutine AP, Kyurkchan AG, Minaev SA. About a universal modification to the method of discrete sources and its application. JQSRT 2003; 79-80: 509-20.

[4] Manenkov SA. Scattering of electromagnetic waves by a nonhomogeneity located in the chiral half-space (Russian) Radiotekh Elektron 2003; 48: 261-67.

[5] Katsenelenbaum BZ, Korshunova EN, Sivov AN and Shatrov AD. Chiral electromagnetic objects (Russian) Usp Fiz Nauk 1997; 167: 1201-1212.

φ, dgrs.	I_E		I_H	
	MGRS	MDSM	MGRS	MDSM
18	$.8490191 \cdot 10^{-1}$	$.8490177 \cdot 10^{-1}$	$.5094825 \cdot 10^{-2}$	$.5094846 \cdot 10^{-2}$
36	.1595335	.1595332	$.6606685 \cdot 10^{-2}$	$.6606729 \cdot 10^{-2}$
54	.1966048	.1966045	$.3500545 \cdot 10^{-2}$	$.3500573 \cdot 10^{-2}$
72	.2096238	.2096239	$.1591314 \cdot 10^{-2}$	$.1591299 \cdot 10^{-2}$
90	.2210460	.2210462	$.1093078 \cdot 10^{-2}$	$.1093052 \cdot 10^{-2}$
108	.2307236	.2307236	$.1236544 \cdot 10^{-2}$	$.1236550 \cdot 10^{-2}$
126	.2235352	.2235349	$.1904224 \cdot 10^{-2}$	$.1904252 \cdot 10^{-2}$
144	.1858746	.1858743	$.2852605 \cdot 10^{-2}$	$.2852611 \cdot 10^{-2}$
162	.1078156	.1078156	$.2306175 \cdot 10^{-2}$	$.2306172 \cdot 10^{-2}$

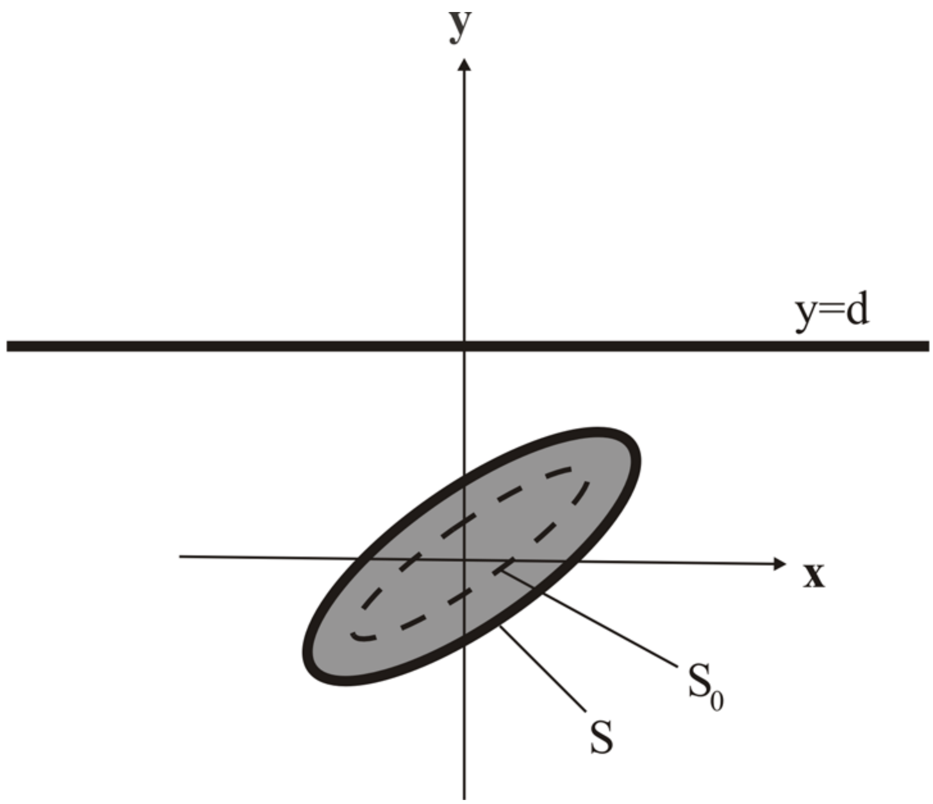


Fig.1.

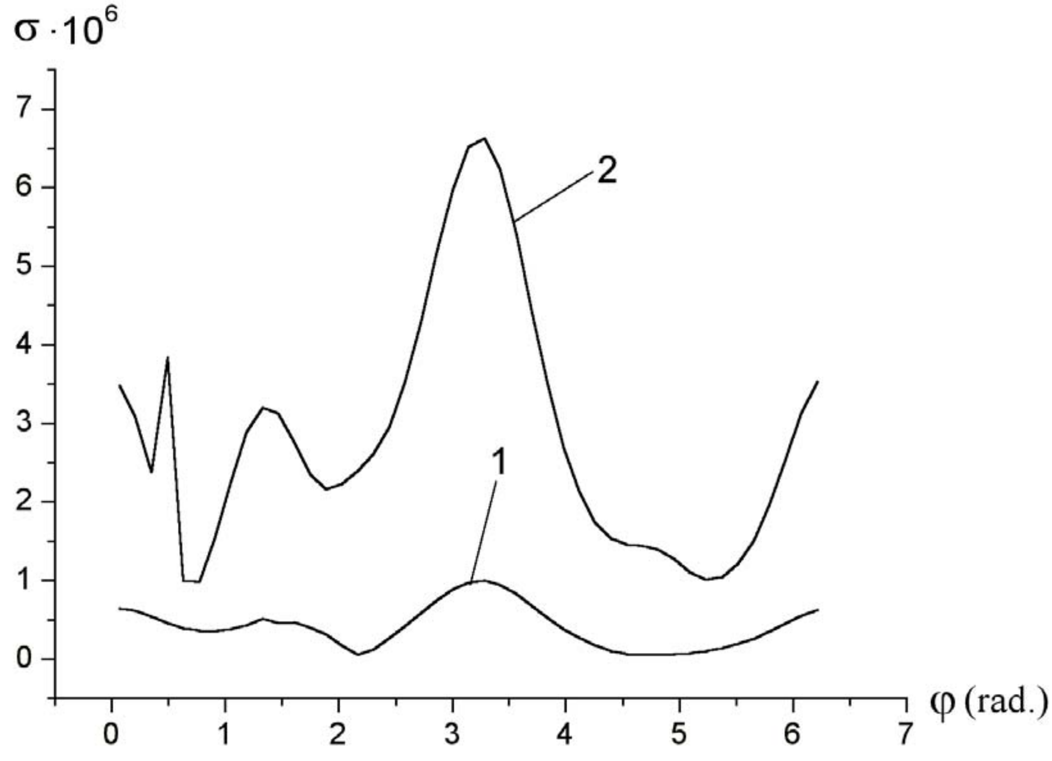


Fig.2.

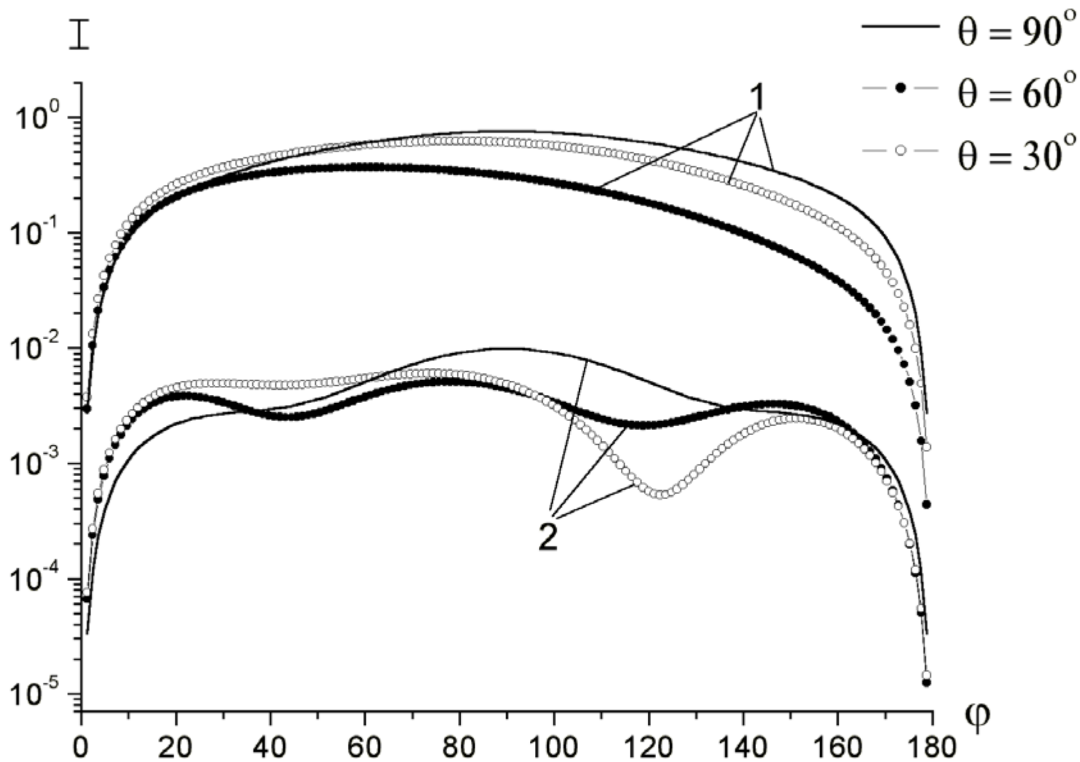

I
θ = 90°
θ = 60°
θ = 30°
1
2
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0 20 40 60 80 100 120 140 160 180
φ

Fig.3.

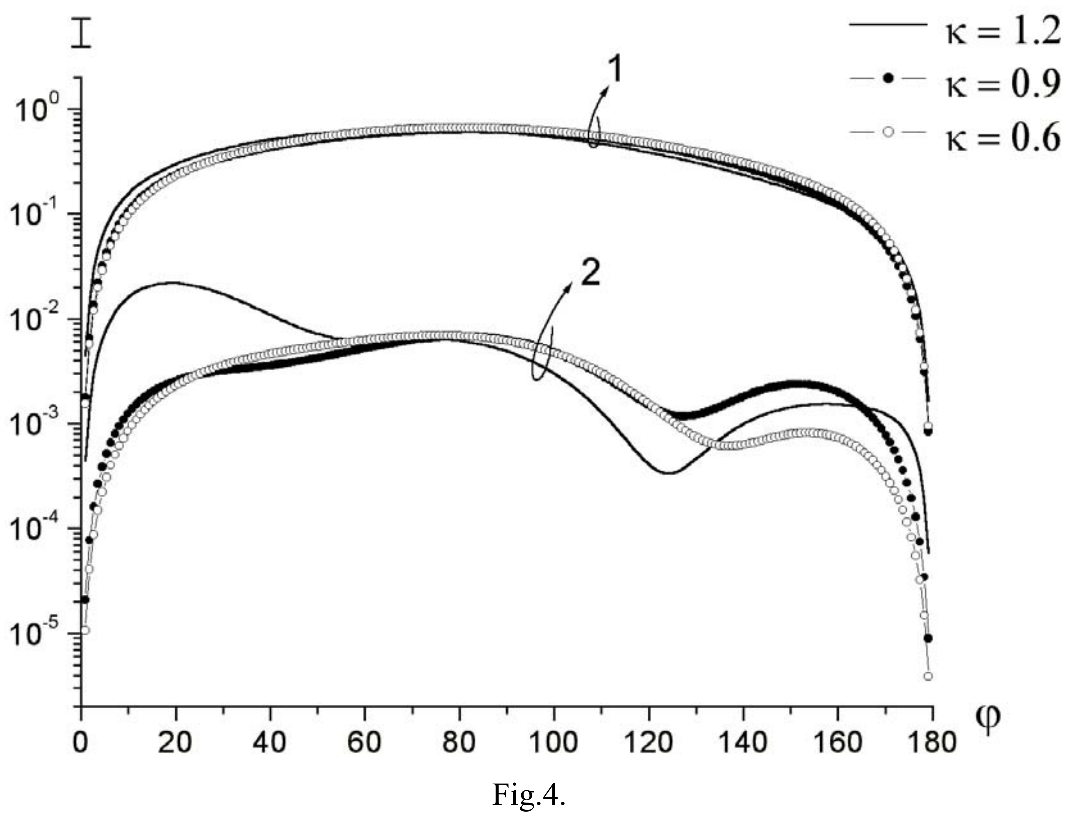
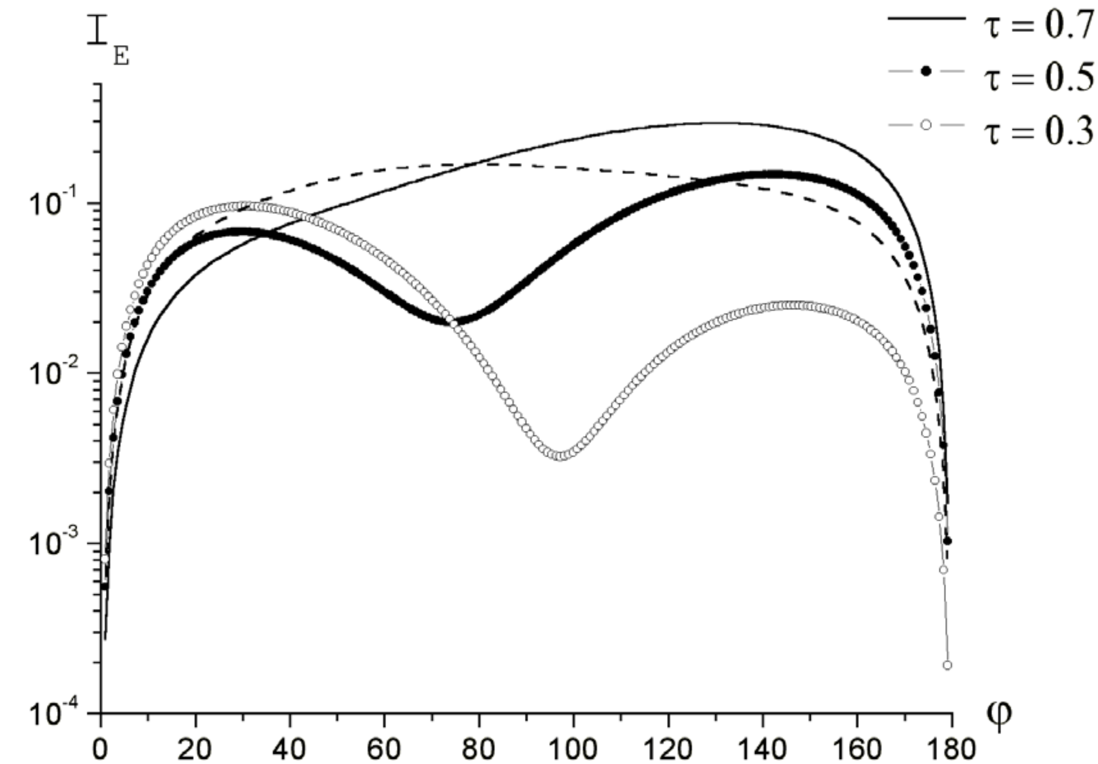
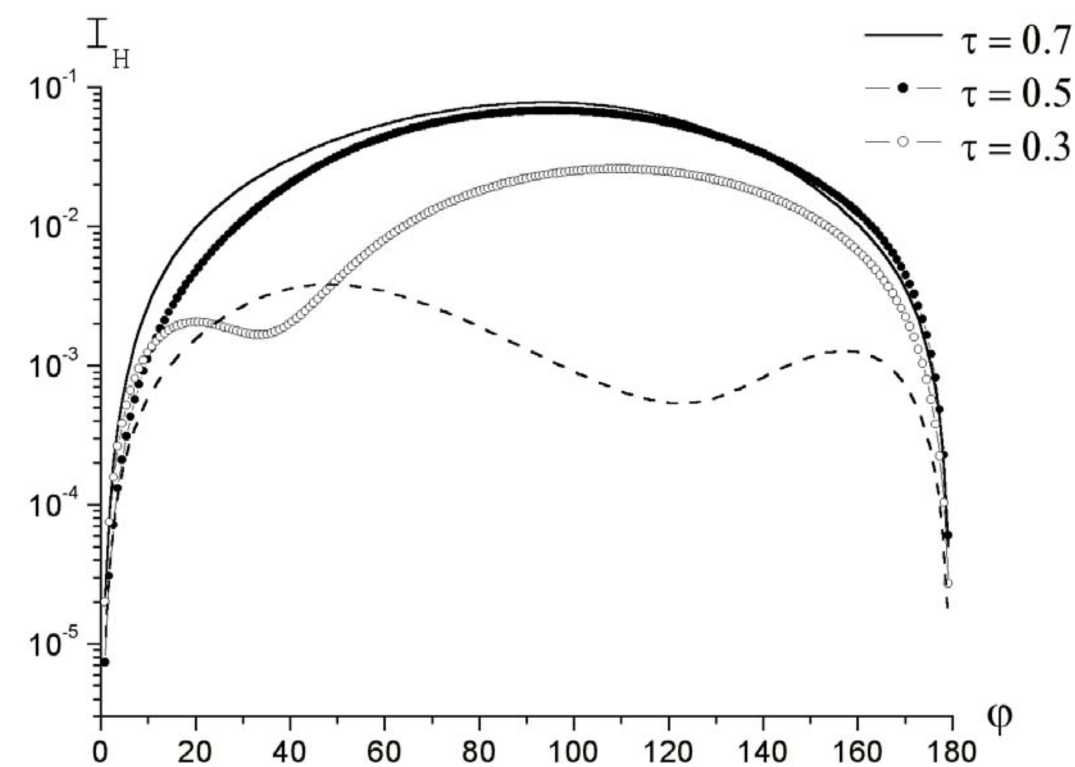

I
κ = 1.2
κ = 0.9
κ = 0.6
1
2
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0 20 40 60 80 100 120 140 160 180
φ

Fig.4.



I
E
τ = 0.7
τ = 0.5
τ = 0.3
10-1
10-2
10-3
10-4
φ
0
20
40
60
80
100
120
140
160
180

Fig.5*a*



I
H
τ = 0.7
τ = 0.5
τ = 0.3
10-1
10-2
10-3
10-4
10-5
φ
0
20
40
60
80
100
120
140
160
180

Fig.5*b*

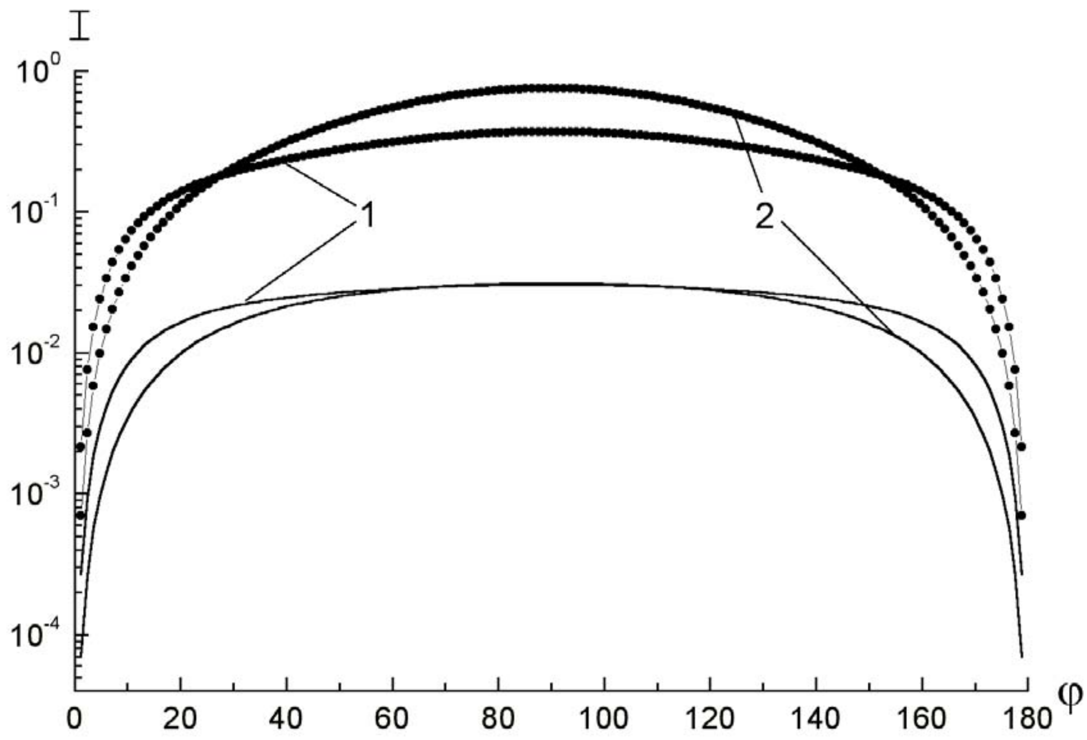

I
10^0
10^-1
10^-2
10^-3
10^-4
1
2
0
20
40
60
80
100
120
140
160
180
φ

Fig.6*a*

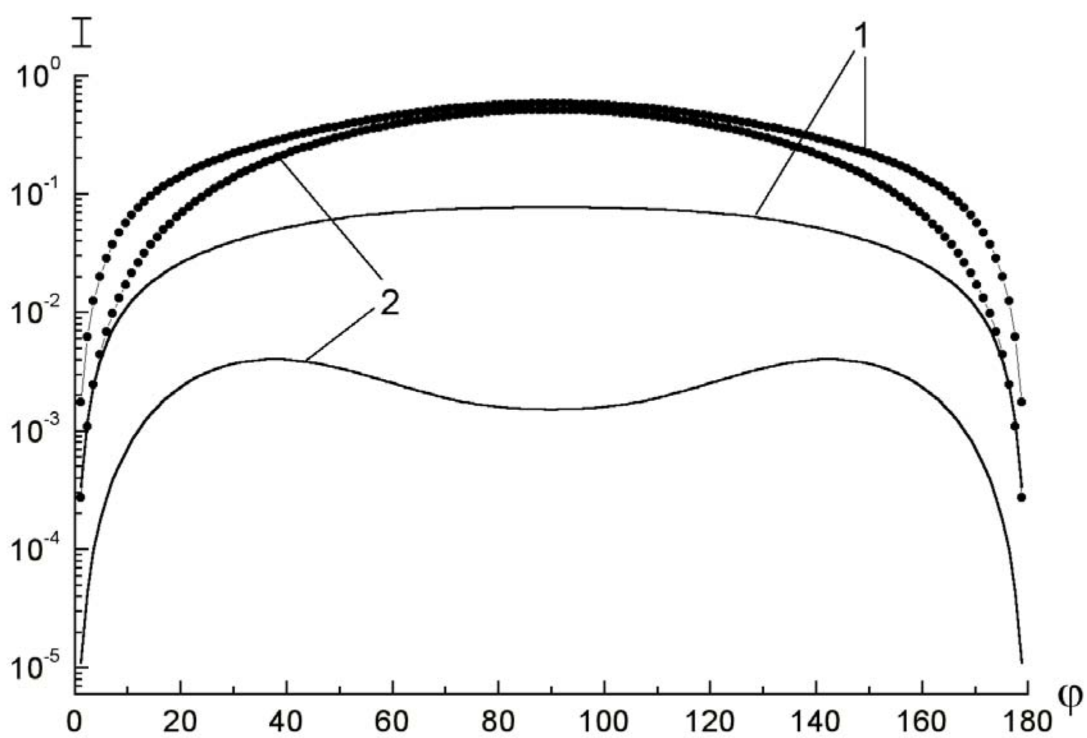
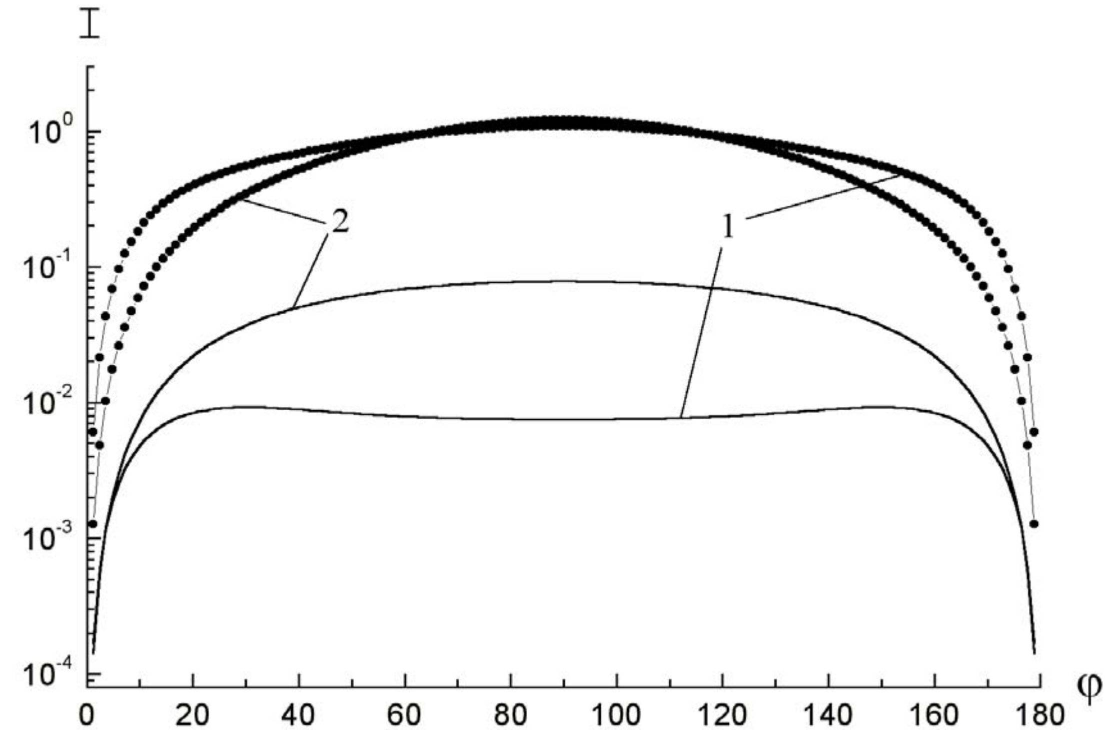
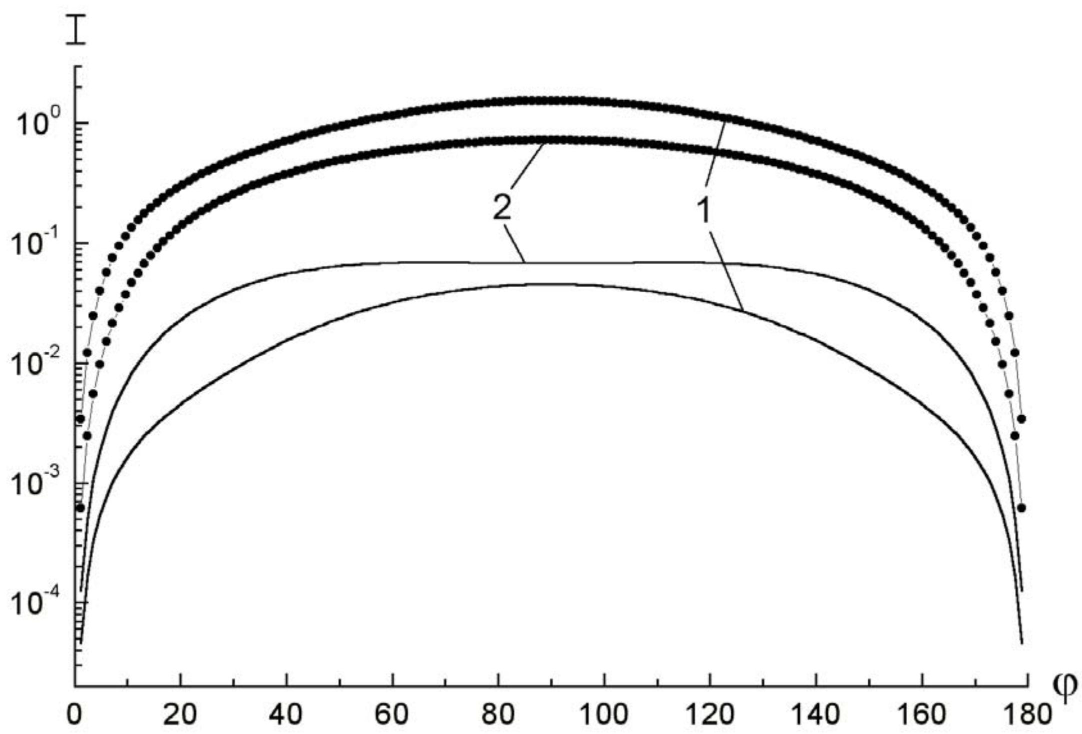

I
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
1
2
0
20
40
60
80
100
120
140
160
180
φ

Fig.6*b*



I
10^0
10^-1
10^-2
10^-3
10^-4
2
1
0
20
40
60
80
100
120
140
160
180
φ

Fig.7*a*



I
10^0
10^-1
10^-2
10^-3
10^-4
2
1
0
20
40
60
80
100
120
140
160
180
φ

Fig.7*b*