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# Null-field method with discrete sources to electromagnetic scattering from composite objects

Adrian Doicu, Thomas Wriedt *

*Institut für Werkstofftechnik, Badgasteiner Strasse 3, 28359 Bremen, Germany*

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**Abstract**

A novel formulation for improving the numerical stability of the null-field method for highly elongated and flattened composite scatterers is presented. The key step in this approach is to approximate the surface current densities by the lowest-order multipoles located in the complex plane. The accuracy of the proposed method is investigated from a numerical point of view. © 2001 Published by Elsevier Science B.V.

*Keywords:* Null-field method; Discrete sources method; Composite scatterers; Lowest-order multipoles; Electromagnetic scattering; Light scattering; Multipoles in complex plane

## 1. Introduction

Electromagnetic scattering from composite scatterers can be computed by using several methods. One of these approaches is the null-field method [1,2]. Ström and Zheng [2] elaborated various null-field formulations using local sources. Their conventional formulation involves the **Q** matrices related to the closed surfaces bounding the homogeneous parts of the composite scatterer. Other formulations use the **Q** matrices related to the interior interface (which is an open surface). For a particular class of elongated composite scatterers this formulation is a useful alternative. Zheng [3] extended the conventional formalism to the case with three or more constituents.

The purpose of the present paper is to develop a null-field formalism using discrete sources. As discrete sources we use the system of distributed lowest-order multipoles. In fact we extend our method for homogeneous scatterers [4] to composite scatterers. As it was shown in the homogeneous case we expect that the use of discrete sources will extend the domain of applicability of the null-field method.

## 2. Basic equations

We consider the generic case of a composite scatterer that consists of two different homogeneous regions $D_1$ and $D_2$. The geometry of the scatterer is shown in Fig. 1. The surface between the domain $D_1$ and $D_2$ is denoted by $S_{12}$. Besides the common origin O, there is one local origin $O_i$ in every homogeneous region $D_i$. The radius vector from the local origin $O_i$ to a point $M$ on the

* Corresponding author. Tel.: +49-421-218-2507; fax: +49-421-218-5378.

*E-mail address:* thw@iwt.uni-bremen.de (T. Wriedt).

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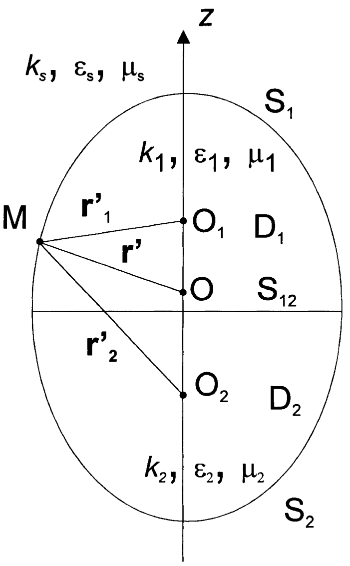


Fig. 1. Geometry of a composite scatterer that consists of two different homogeneous regions.

surface is denoted by $\mathbf{r}_i'$ and from the common origin O to the point $M$ by $\mathbf{r}'$. The wave number of the region $D_i$ is $k_i = k(\varepsilon_i\mu_i)^{1/2}$, while the wave number for the free space is $k_s = k(\varepsilon_s\mu_s)^{1/2}$, where $k = \omega/c$.

Let us rewrite the basic equations of the conventional null-field method in a slightly different form as given by Ström and Zheng [2]. By considering the null-field conditions for the total electric field within $D_i$, we obtain the following set of integral equations for the surface current densities:

$$\sum_{j=1}^{2}\int_{S_j\cup S_{12}}\left[[\mathbf{e}_j^{\mathcal{N}}(\mathbf{r}_j') - \mathbf{e}_0(\mathbf{r}_j')]\begin{pmatrix}\mathbf{N}_{-mn}^3(k_s\mathbf{r}_i')\\ \mathbf{M}_{-mn}^3(k_s\mathbf{r}_i')\end{pmatrix} + \mathrm{j}\sqrt{\frac{\mu_s}{\varepsilon_s}}[\mathbf{h}_j^{\mathcal{N}}(\mathbf{r}_j') - \mathbf{h}_0(\mathbf{r}_j')]\begin{pmatrix}\mathbf{N}_{-mn}^3(k_s\mathbf{r}_i')\\ \mathbf{M}_{-mn}^3(k_s\mathbf{r}_i')\end{pmatrix}\right]\mathrm{d}S(\mathbf{r}_j') = 0, \tag{1}$$

where $i = 1,\ 2,\ m = -M,\dots,M$ and $n = |m|,\dots,N$. Here, we denote by $\mathbf{e}_j$, $\mathbf{h}_j$ the surface current densities on the closed surface $S_j \cup S_{12}$ and by $\mathbf{e}_0$, $\mathbf{h}_0$ the tangential components of the incident electric and magnetic field, respectively. Note, the continuity conditions $\mathbf{e}_1 = \mathbf{e}_2$ and $\mathbf{h}_1 = \mathbf{h}_2$ on $S_{12}$. The index $\mathcal{N}$ is a complex index incorporating $M$ and $N$. In the conventional method the integrals containing the tangential components of the incident field are identified as the expansion coefficients of the external excitation in terms of the regular spherical vector wave functions.

An approximate solution of the scattering problem can be obtained by approximating the surface current densities by the complete set of tangential, single spherical coordinate vector wave functions

$$\begin{pmatrix}\mathbf{e}_j^{\mathcal{N}}(\mathbf{r}_j')\\ \mathbf{h}_j^{\mathcal{N}}(\mathbf{r}_j')\end{pmatrix} = \sum_{m=-M}^{M}\sum_{n=|m|}^{N} a_{mn}^{j\mathcal{N}}\begin{pmatrix}\mathbf{n}\times\mathbf{M}_{mn}^1(k_j\mathbf{r}_j')\\ -\mathrm{j}\sqrt{\frac{\varepsilon_j}{\mu_j}}\mathbf{n}\times\mathbf{N}_{mn}^1(k_j\mathbf{r}_j')\end{pmatrix} + b_{mn}^{j\mathcal{N}}\begin{pmatrix}\mathbf{n}\times\mathbf{N}_{mn}^1(k_j\mathbf{r}_j')\\ -\mathrm{j}\sqrt{\frac{\varepsilon_j}{\mu_j}}\mathbf{n}\times\mathbf{M}_{mn}^1(k_j\mathbf{r}_j')\end{pmatrix}. \tag{2}$$

Once the surface current densities are determined the approximate solution of the scattered field can be obtained by using the Stratton–Chu representation formulas.

The above formalism can be modified by replacing the set of single spherical coordinate vector wave functions $\{\mathbf{M}_{mn}^{1,3}(k\mathbf{r}_i),\ \mathbf{N}_{mn}^{1,3}(k\mathbf{r}_i)\}_{m\in Z, n=|m|,\dots}$ by the system lowest-order spherical vector wave functions $\{\mathbf{M}_{m,|m|+l}^3[k(\mathbf{r}_i - z_p^i\mathbf{e}_3)],\ \mathbf{N}_{m,|m|+l}^3[k(\mathbf{r}_i - z_p^i\mathbf{e}_3)]\}_{m\in Z, p=1,2,\dots}$, where $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ are the unit vectors in Cartesian coordinates, $(z_p^i)_{p=1,2,\dots}$ is a dense sequence of points on a segment $\Gamma_z^i$ of the $z$-axis, $\Gamma_z^i \subset D_i$, and $l = 1$ if $m = 0$ and $l = 0$ otherwise. By employing the same arguments as given by Doicu et al. [5] one can show that the set of integral equations

$$\sum_{j=1}^{2}\int_{S_j\cup S_{12}}\left[[\mathbf{e}_j^{\mathcal{N}}(\mathbf{r}_j') - \mathbf{e}_0(\mathbf{r}_j')]\begin{pmatrix}\mathbf{N}_{-m,|m|+l}^3[k_s(\mathbf{r}_i' - z_p^i\mathbf{e}_3)]\\ \mathbf{M}_{-m,|m|+l}^3[k_s(\mathbf{r}_i' - z_p^i\mathbf{e}_3)]\end{pmatrix} + \mathrm{j}\sqrt{\frac{\mu_s}{\varepsilon_s}}[\mathbf{h}_j^{\mathcal{N}}(\mathbf{r}_j') - \mathbf{h}_0(\mathbf{r}_j')]\begin{pmatrix}\mathbf{N}_{-m,|m|+l}^3[k_s(\mathbf{r}_i' - z_p^i\mathbf{e}_3)]\\ \mathbf{M}_{-m,|m|+l}^3[k_s(\mathbf{r}_i' - z_p^i\mathbf{e}_3)]\end{pmatrix}\right] \times \mathrm{d}S(\mathbf{r}_j') = 0, \tag{3}$$

with $i = 1,\ 2,\ m = -M,\dots,M$ and $p = 1,\dots,N$, guarantees the null-field condition for the total electric field within $D_i$, $i = 1, 2$. The surface current densities are now approximated by the complete system of the tangential components of the lowest-order spherical vector wave functions, i.e.

$$\begin{pmatrix} \mathbf{e}_j^{\mathcal{N}}(\mathbf{r}'_j) \\ \mathbf{h}_j^{\mathcal{N}}(\mathbf{r}'_j) \end{pmatrix} = \sum_{m=-M}^{M} \sum_{p=1}^{N} a_{mp}^{j\mathcal{N}} \begin{pmatrix} \mathbf{n} \times \mathbf{M}^1_{m,|m|+l}[k_j(\mathbf{r}'_j - z_p^j \mathbf{e}_3)] \\ -\mathrm{j}\sqrt{\frac{\varepsilon_j}{\mu_j}}\mathbf{n} \times \mathbf{N}^1_{m,|m|+l}[k_j(\mathbf{r}'_j - z_p^j \mathbf{e}_3)] \end{pmatrix} + b_{mp}^{j\mathcal{N}} \begin{pmatrix} \mathbf{n} \times \mathbf{N}^1_{m,|m|+l}[k_j(\mathbf{r}'_j - z_p^j \mathbf{e}_3)] \\ -\mathrm{j}\sqrt{\frac{\varepsilon_j}{\mu_j}}\mathbf{n} \times \mathbf{M}^1_{m,|m|+l}[k_j(\mathbf{r}'_j - z_p^j \mathbf{e}_3)] \end{pmatrix}. \tag{4}$$

We note that the above formalism can be extended in a simple manner to the case with three or more constituents.

The advantage of using the system of lowest-order spherical vector wave functions as a system of discrete sources is that for axisymmetric scatterers the problem decouples over the azimuthal modes $m$. In the case of prolate scatterers, the choice of the multipoles on the axis of symmetry adequately describes the particle geometry. This arrangement is not suitable for oblate scatterers. In this case the procedure of analytic continuation of the representation of the spherical vector-wave functions onto the complex plane along the source coordinate $z$ is used [5]. Essentially, this procedure gives the possibility of correlating the position of the support of discrete sources with the singularities of the analytic continuation of the scattered field inside the composite particle. The spherical vector wave functions can be expressed in terms of the coordinates of the source point $\widehat{\mathbf{z}} \in \widehat{\Sigma}$ and the observation point $\eta \in \Sigma$ by

$$\mathbf{M}^{1,3}_{mn}(k\mathbf{r}) = z_n^{1,3}(kR)\left\{\mathrm{j}m\frac{P_n^{|m|}(\cos\widehat{\theta})}{\sin\widehat{\theta}}[\sin(\theta-\widehat{\theta})\mathbf{e}_r + \cos(\theta-\widehat{\theta})\mathbf{e}_\theta] - \frac{\mathrm{d}P_n^{|m|}(\cos\widehat{\theta})}{\mathrm{d}\widehat{\theta}}\mathbf{e}_\varphi\right\}\mathrm{e}^{\mathrm{j}m\varphi} \tag{5}$$

and

$$\mathbf{N}^{1,3}_{mn}(k\mathbf{r}) = \left\{n(n+1)\frac{z_n^{1,3}(kR)}{kR}P_n^{|m|}(\cos\widehat{\theta})[\cos(\theta-\widehat{\theta})\mathbf{e}_r - \sin(\theta-\widehat{\theta})\mathbf{e}_\theta] + \frac{1}{kR}\frac{\mathrm{d}}{\mathrm{d}R}[Rz_n^{1,3}(kR)] \times \frac{\mathrm{d}P_n^{|m|}(\cos\widehat{\theta})}{\mathrm{d}\widehat{\theta}}[\sin(\theta-\widehat{\theta})\mathbf{e}_r + \cos(\theta-\widehat{\theta})\mathbf{e}_\theta] + \mathrm{j}m\frac{1}{kR}\frac{\mathrm{d}}{\mathrm{d}R}[Rz_n^{1,3}(kR)] \times \frac{P_n^{|m|}(\cos\widehat{\theta})}{\sin\widehat{\theta}}\mathbf{e}_\varphi\right\}\mathrm{e}^{\mathrm{j}m\varphi}, \tag{6}$$

where $z_n^{1,3}(kR)$ denotes the spherical Bessel functions or the spherical Hankel functions, respectively, $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\varphi)$ are the unit vectors in spherical coordinates, $P_n^{|m|}$ are the Legendre functions and

$$R^2 = \rho^2 + (z - \widehat{\mathbf{z}})^2, \quad \sin\widehat{\theta} = \frac{\rho}{R}, \quad \cos\widehat{\theta} = \frac{z - \widehat{\mathbf{z}}}{R}.$$

The complex plane $\widehat{\Sigma} = \{\widehat{\mathbf{z}} = (\mathrm{Re}\,\widehat{\mathbf{z}}, \mathrm{Im}\,\widehat{\mathbf{z}})/\mathrm{Re}\,\widehat{\mathbf{z}}, \mathrm{Im}\,\widehat{\mathbf{z}} \in \mathbf{R}\}$ is defined such that real axis $\mathrm{Re}\,\widehat{\mathbf{z}}$ coincides with the $z$-axis of the azimuthal plane $\varphi = \mathrm{const}$, $\Sigma = \{\eta = (\rho, z)/\rho \geqslant 0,\ z \in \mathbf{R}\}$. The region in which the spherical vector wave functions are analytical with respect to the variable $\widehat{\mathbf{z}}$ is the domain $\widehat{D}$ whose boundary coincides with the image of the generator of revolution. Note that a point $\widehat{\mathbf{z}} \in \widehat{\Sigma}$ is called the image of a point $\eta \in \Sigma$ if $R^2_{\eta\widehat{\mathbf{z}}} = 0$. In this case, for a sequence of poles distributed symmetrical with respect to the $\mathrm{Re}\,\widehat{\mathbf{z}}$-axis and having at least two limit points in $\widehat{D}$, we can construct an approximate solution by using Eqs. (3) and (4).

## 3. Numerical results

The formulation presented in Section 2 has been implemented in a computer program. In this sections we present computer results for prolate and oblate composite spheroids. The scatterer coordinate system is denoted by O$xyz$ with the $z$-axis directed along the symmetry axis of the scatterer. For simplicity we assume that the incident wave is a plane wave travelling along the symmetry axis of the scatterer. The polarization direction encloses an angle $\alpha_{\mathrm{pol}}$ with the $x$-axis. The composite scatterer consists of two half-spheroids with refractive

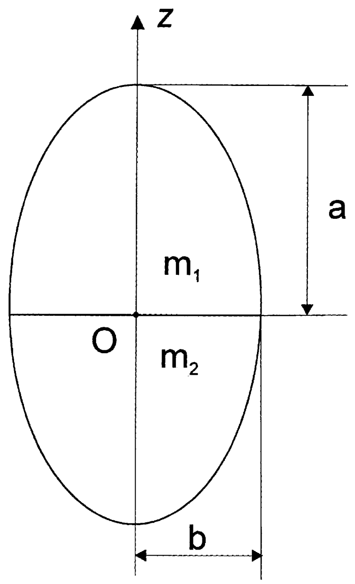


Fig. 2. Illustration of the composite scatterer considered in our simulations.

indices $m_1 = 1.5$ and $m_2 = 1.3$. The semiaxes of the spheroid are denoted by $a$ and $b$, as it shown in Fig. 2. Specifically, the differential scattering cross section (DSCS) normalized by $\pi a^2$ will be computed in the azimuthal plane $\varphi = 0°$ for parallel and perpendicular polarization of the incident wave.

In our first example we consider an prolate composite scatterer with $k_s a = 10$ and $k_s b = 1$. The results corresponding to the null-field method with localized and distributed sources are shown in Fig. 3. For this case we used a collection of multipoles located on the symmetry axis of the scatterer. The complete agreement between the curves serves as an evidence for the accuracy of the proposed methods. In Fig. 4 we plot the DSCS for a composite scatterer with $k_s a = 10$ and $k_s b = 0.25$. Only the results corresponding to the null-field method with distributed sources are shown since the null-field method with localized sources fails to converge.

The next problem is that of an oblate spheroid with $k_s a = 5$ and $k_s b = 8$. The results plotted in Fig. 5 clearly demonstrate that no significant differences exist between the scattering diagrams. For

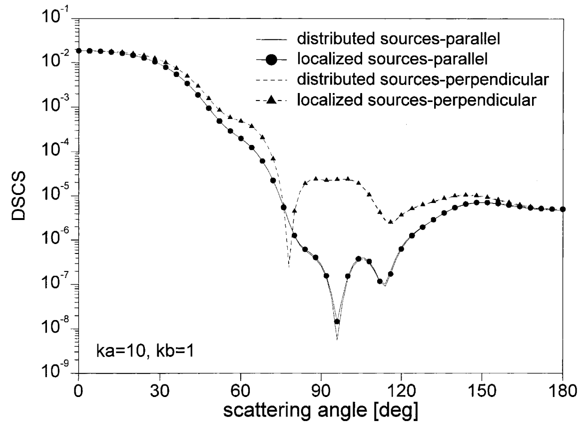


Fig. 3. DSCS for a composite prolate scatterer with $k_s a = 10$ and $k_s b = 1$. The refractive indices of the homogeneous parts are $m_1 = 1.5$ and $m_2 = 1.3$. The results are computed with the null-field method with localized and distributed sources.

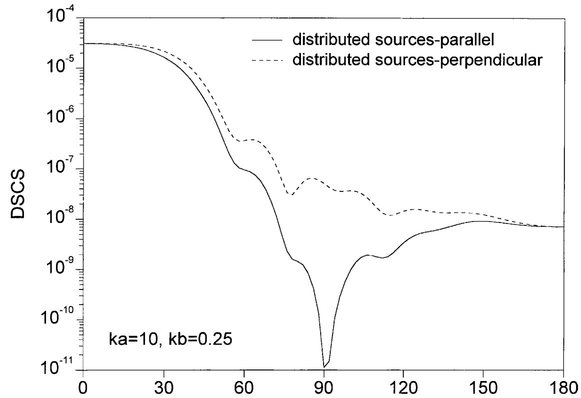


Fig. 4. DSCS for a composite prolate scatterer with $k_s a = 10$ and $k_s b = 0.25$. The refractive indices of the homogeneous parts are $m_1 = 1.5$ and $m_2 = 1.3$. The results are computed with the null-field method with distributed sources.

this type of scatterer we choose the multipoles in the complex plane, on two lines which are parallel to the imaginary axis. The position of the distributed sources is shown in Fig. 6. Plots of the differential scattering cross section for an oblate spheroid with $k_s a = 3$ and $k_s b = 8$ are shown in Fig. 7. As before, the null-field method with localized sources does not converge for this geometry.

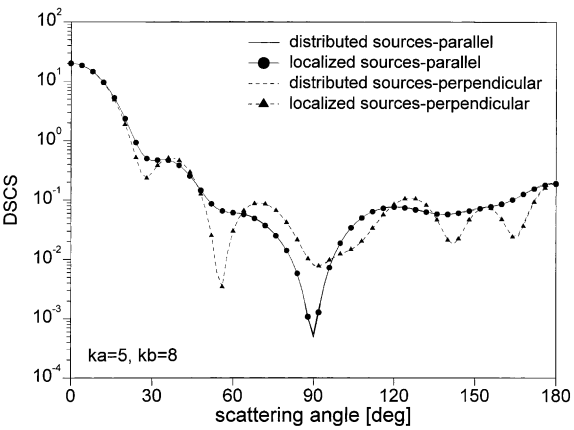


Fig. 5. DSCS for a composite oblate scatterer with $k_s a = 5$ and $k_s b = 8$. The refractive indices of the homogeneous parts are $m_1 = 1.5$ and $m_2 = 1.3$. The results are computed with the null-field method with localized and distributed sources.

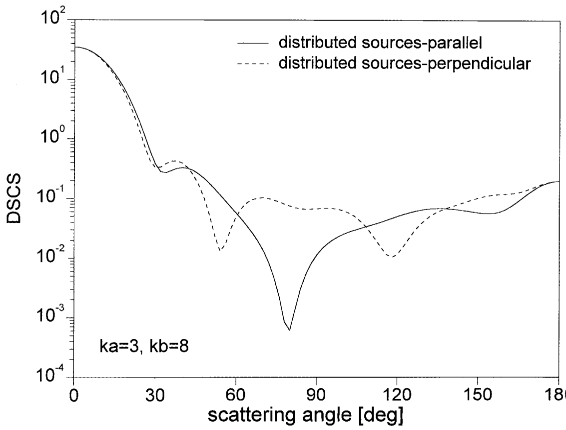


Fig. 7. DSCS for a composite oblate scatterer with $k_s a = 3$ and $k_s b = 8$. The refractive indices of the homogeneous parts are $m_1 = 1.5$ and $m_2 = 1.3$. The results are computed with the null-field method with distributed sources.

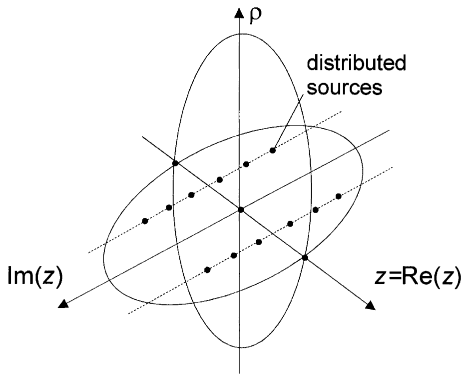


Fig. 6. Position of distributed sources in the complex plane.

## 4. Conclusions

A novel formulation of the null-field method with distributed sources for composite scatterers has been proposed. In this method we choose the tangential components of the lowest-order multipoles with different origins as a complete system of functions on the particle surface. The advantage of the proposed approach over the conventional method concerning the applicability to highly elongated and flattened scatterers have been demonstrated for a number of examples.

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