



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Optics Communications 213 (2002) 21–25

OPTICS
COMMUNICATIONS

www.elsevier.com/locate/optcom

Null-field method with circularly distributed spherical vector wave functions

Adrian Doicu*

DLR-German Aerospace Center, Remote Sensing Technology Institute, Oberpfaffenhofen, 82234 Wessling, Germany

Received 29 July 2002; received in revised form 9 September 2002; accepted 15 September 2002

Abstract

A novel formulation for improving the numerical stability of the null-field method for highly flattened particles is presented. The key step in this approach is to approximate the surface current densities by circularly distributed spherical vector wave functions. The circularly distributed spherical vector wave functions are obtained by integrating the spherical vector wave functions with a shifted origin over the azimuthal angle of the source point. The accuracy of the proposed method is investigated from a numerical point of view.

© 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

One of the most powerful method for computing nonspherical light scattering using localized spherical vector wave functions is the null-field method [1]. However, for particles with extreme geometries the single spherical coordinate based null-field method fails to converge. A number of modifications to the conventional null-field method have been suggested to improve the numerical stability. One of this formal modification is the null-field method with discrete sources [2]. Essentially, this method entails the use of a number of elementary sources to approximate the surface current densities. For strong deformed particles distributed spherical vector wave functions are

adequate. In the case of highly elongated spheroids the multipoles are chosen on the axis of symmetry, while in the case of flattened spheroids the procedure of analytic continuation of the representation of the spherical vector wave functions in the complex plane (along the symmetry axis) is required. One reason for using sources distributed in the complex plane is that the resulting system of vector functions is orthogonal with respect to the azimuthal angle. In this case, the scattering problem of an axisymmetric particle can be reduced to a sequence of subproblems for each azimuthal mode.

The aim of this paper is to present a system of functions which is orthogonal with respect to the azimuthal angle and is suitable for analyzing the scattering by flattened particles. The new system is the system of circularly distributed spherical vector wave functions. A similar system of functions was discussed by Leuchtman [3] and

* Tel.: +49-8153-28-3015; fax: +49-8153-28-1446.

E-mail address: adrian.doicu@dlr.de.

Hafner and Bomholt [4] in the framework of the multiple multipole method. The so-called curved line multipoles or ringmultipoles used in the multiple multipole method are a superposition of z -shaped currents (current segments located parallel to the z -axis) with alternating sign and distributed on a circle. Here, we use a different approach. We integrate the spherical vector wave functions with a shifted origin over the azimuthal angle of the source point and obtain a system of vector functions which is complete on the particle surface and orthogonal with respect to the azimuthal angle.

2. Basic equations

Let us consider a bounded domain D_i (of class \mathbb{C}^2) with boundary S and exterior D_s . The wave number of the region D_i is $k_i = k\sqrt{\varepsilon_i\mu_i}$, while the wave number of the region D_s is $k_s = k\sqrt{\varepsilon_s\mu_s}$, where $k = \omega/c$. Let us denote by $(r_0, \theta_0, \varphi_0)$ the spherical coordinates of the source point \mathbf{r}_0 , while (r, θ, φ) stand for the spherical coordinates of the observation point \mathbf{r} . Let us consider the set of spherical vector wave functions $\mathbf{M}_{mn}^{1,3}(\mathbf{r} - \mathbf{r}_0)$ and $\mathbf{N}_{mn}^{1,3}(\mathbf{r} - \mathbf{r}_0)$, where $n = 1, 2, \dots$, and $m = -n, \dots, n$. Here, \mathbf{M}_{mn}^1 and \mathbf{N}_{mn}^1 is an entire solution to the Maxwell equations and \mathbf{M}_{mn}^3 and \mathbf{N}_{mn}^3 is a radiating solution to the Maxwell equations in $\mathbb{R}^3 - \{\mathbf{r}_0\}$. The system of vector functions

$$\left\{ \left(\begin{array}{c} \mathbf{n} \times \mathbf{M}_{mn}^{1,3} \\ -j\sqrt{\frac{\varepsilon_{i,s}}{\mu_{i,s}}}\mathbf{n} \times \mathbf{N}_{mn}^{1,3} \end{array} \right), \left(\begin{array}{c} \mathbf{n} \times \mathbf{N}_{mn}^{1,3} \\ -j\sqrt{\frac{\varepsilon_{i,s}}{\mu_{i,s}}}\mathbf{n} \times \mathbf{M}_{mn}^{1,3} \end{array} \right) \right\} \quad (1)$$

with $n = 1, 2, \dots$, and $m = -n, \dots, n$ is complete in $\mathcal{L}_{\tan}^2(S) = L_{\tan}^2(S) \times L_{\tan}^2(S)$. In Eq. (1), \mathbf{n} stands for unit outward normal to S . The radiating spherical vector wave functions are defined with respect to the wave number k_s , while the regular spherical vector wave functions are defined with respect to the wave number k_i . For a proof we refer to [2]. Using the addition theorem for spherical vector wave functions under coordinate translation and the integral representation of the translation coefficients we find that

$$\mathbf{M}_{mn}^{1,3}(\mathbf{r} - \mathbf{r}_0) = \sum_l \mathbf{m}_{lmn}^{1,3}(\eta, \eta_0) e^{-j(l-m)\varphi_0} e^{jl\varphi} \quad (2)$$

and

$$\mathbf{N}_{mn}^{1,3}(\mathbf{r} - \mathbf{r}_0) = \sum_l \mathbf{n}_{lmn}^{1,3}(\eta, \eta_0) e^{-j(l-m)\varphi_0} e^{jl\varphi}, \quad (3)$$

where $\eta = (r, \theta)$ and $\eta_0 = (r_0, \theta_0)$. Note that the Fourier components $\mathbf{m}_{lmn}^{1,3}$ and $\mathbf{n}_{lmn}^{1,3}$ are expressed in terms of the spherical unit vectors $\mathbf{e}_r, \mathbf{e}_\theta$ and \mathbf{e}_φ corresponding to \mathbf{r} . We define the set of circularly distributed spherical vector wave functions by

$$\begin{aligned} \mathcal{M}_{lmn}^{1,3}(\mathbf{r}) &= \mathbf{m}_{lmn}^{1,3}(\eta, \eta_0) e^{jl\varphi} \\ &= \frac{1}{2\pi} \int_0^{2\pi} \mathbf{M}_{lmn}^{1,3}(\mathbf{r} - \mathbf{r}_0) e^{j(l-m)\varphi_0} d\varphi_0 \end{aligned} \quad (4)$$

and

$$\begin{aligned} \mathcal{N}_{lmn}^{1,3}(\mathbf{r}) &= \mathbf{n}_{lmn}^{1,3}(\eta, \eta_0) e^{jl\varphi} \\ &= \frac{1}{2\pi} \int_0^{2\pi} \mathbf{N}_{lmn}^{1,3}(\mathbf{r} - \mathbf{r}_0) e^{j(l-m)\varphi_0} d\varphi_0. \end{aligned} \quad (5)$$

Then, it is trivial to show that the system of vector functions

$$\left\{ \left(\begin{array}{c} \mathbf{n} \times \mathcal{M}_{lmn}^{1,3} \\ -j\sqrt{\frac{\varepsilon_{i,s}}{\mu_{i,s}}}\mathbf{n} \times \mathcal{N}_{lmn}^{1,3} \end{array} \right), \left(\begin{array}{c} \mathbf{n} \times \mathcal{N}_{lmn}^{1,3} \\ -j\sqrt{\frac{\varepsilon_{i,s}}{\mu_{i,s}}}\mathbf{n} \times \mathcal{M}_{lmn}^{1,3} \end{array} \right) \right\} \quad (6)$$

with $l \in \mathbb{Z}$, $n = 1, 2, \dots$, and $m = -n, \dots, n$ is complete in $\mathcal{L}_{\tan}^2(S) = L_{\tan}^2(S) \times L_{\tan}^2(S)$. Similarly, one can show that the set of integral equations

$$\begin{aligned} \int_S \left\{ [\mathbf{e}(\mathbf{r}') - \mathbf{e}_0(\mathbf{r}')] \left(\begin{array}{c} \mathcal{N}_{-lmn}^3(\mathbf{r}') \\ \mathcal{M}_{-lmn}^3(\mathbf{r}') \end{array} \right) \right. \\ \left. + j\sqrt{\frac{\mu_s}{\varepsilon_s}} [\mathbf{h}(\mathbf{r}') - \mathbf{h}_0(\mathbf{r}')] \left(\begin{array}{c} \mathcal{M}_{-lmn}^3(\mathbf{r}') \\ \mathcal{N}_{-lmn}^3(\mathbf{r}') \end{array} \right) \right\} dS(\mathbf{r}') = 0 \end{aligned} \quad (7)$$

with $l \in \mathbb{Z}$, $n = 1, 2, \dots$, and $m = -n, \dots, n$, guarantees the null-field condition of the total electric field within D_i . In Eq. (7), \mathbf{e} and \mathbf{h} stand for the surface current densities on the surface S , and \mathbf{e}_0 and \mathbf{h}_0 are the tangential components of the incident electric and magnetic field, respectively. An approximate solution to the scattering problem can be obtained by approximating the surface current densities by the complete system of circu-

larly distributed spherical vector wave functions, i.e.,

$$\begin{pmatrix} \mathbf{e}_{\mathcal{N}}(\mathbf{r}') \\ \mathbf{h}_{\mathcal{N}}(\mathbf{r}') \end{pmatrix} = \sum_{l=-L}^L \sum_{m=-M}^M \sum_{n=\max(1,|m|)}^N a_{lmn}^{\mathcal{N}} \begin{pmatrix} \mathbf{n} \times \mathcal{M}_{lmn}^1(\mathbf{r}') \\ -j\sqrt{\frac{\epsilon_i}{\mu_i}} \mathbf{n} \times \mathcal{N}_{lmn}^1(\mathbf{r}') \end{pmatrix} + b_{lmn}^{\mathcal{N}} \begin{pmatrix} \mathbf{n} \times \mathcal{N}_{lmn}^1(\mathbf{r}') \\ -j\sqrt{\frac{\epsilon_i}{\mu_i}} \mathbf{n} \times \mathcal{M}_{lmn}^1(\mathbf{r}') \end{pmatrix}. \quad (8)$$

Here \mathcal{N} is a complex index incorporating L , M and N . Once the surface current densities are determined the approximate scattered field can be obtained by using the Stratton–Chu representation formulas. The circularly distributed spherical vector wave functions have an $\exp(jl\varphi)$ -dependence. Consequently, the determination of the surface current densities for an axisymmetric particle can be broken up into a sequence of decoupled problems over each azimuthal mode and the surface integrals simplify to line integrals with respect to the polar angle θ . However, this last simplification is only apparent since the Fourier components must be computed by numerical integration and this process can be time consuming. Since the argument of the spherical Hankel functions is $k_s|\mathbf{r} - \mathbf{r}_0|$ we expect to obtain a stable approximation of the solution especially for highly flattened particles. In this case, we can choose \mathbf{r}_0 such that the argument of the Hankel functions does not vary significantly during the integration process (as in the case of a localized source).

In practice, the system of circularly distributed spherical vector wave functions can also be used in combination with other discrete sources to obtain a stable approximation of the solution. For instance, the surface current densities can be approximated by a linear combination of circularly distributed and localized spherical vector wave functions. Accordingly, the null-field equations will be expressed in terms of circularly distributed and localized spherical vector wave functions.

A similar system of vector functions can be constructed as

$$\begin{aligned} \mathcal{M}_{lmn}^{1,3}(\mathbf{r}) &= \nabla \left[\frac{1}{2\pi} \int_S u_{mn}^{1,3}(\mathbf{r} - \mathbf{r}_0) e^{j(l-m)\varphi_0} d\varphi_0 \right] \times \mathbf{r}, \\ \mathcal{N}_{lmn}^{1,3}(\mathbf{r}) &= \frac{1}{k_{i,s}} \nabla \times \mathcal{M}_{lmn}^{1,3}(\mathbf{r}). \end{aligned} \quad (9)$$

Although, this system of vector functions is also complete in $\mathcal{L}_{\tan}^2(S)$ we will restrict our numerical analysis to the system given by Eqs. (4) and (5).

3. Numerical results

In this section, we present computer results for oblate and prolate spheroids. The scatterer coordinate system is denoted by $Oxyz$ with the z -axis directed along the symmetry axis of the scatterer. The semiaxes of the spheroid are denoted by a and b , with a being along the symmetry axis of the scatterer. For simplicity, we assume that the incident wave is a plane wave traveling along the symmetry axis of the scatterer. In this case only the azimuthal modes $l = \pm 1$ are required for the solution of the scattering problem. The polarization angle of the incident wave is 45° . The refractive index of the particle was chosen as 1.5. Specifically, the differential scattering cross-section (DSCS) normalized by πl^2 will be computed in the azimuthal plane $\varphi = 0^\circ$. Here, l is a normalization length which was chosen as $k_s l = 10$.

In our first example, we consider a prolate spheroid with $k_s a = 8$ and $k_s b = 4$. The results corresponding to the null-field method with circularly distributed spherical sector wave functions together with the results corresponding to the null-field method with localized spherical vector wave functions are shown in Fig. 1. The sources were distributed in the xy -plane, on a circle of radius $k_s r_0 = 1$. In Fig. 2, we plot the differential scattering cross-section for an oblate particle with $k_s a = 4$ and $k_s b = 8$. In this case, the sources were distributed in the xy -plane, on a circle of radius $k_s r_0 = 4$. The complete agreement between the curves serves as an evidence for the accuracy of the proposed method.

The next problem is that of an oblate spheroid with $k_s a = 1$ and $k_s b = 8$. The results corresponding to the null-field method with circularly distributed spherical vector wave functions and a

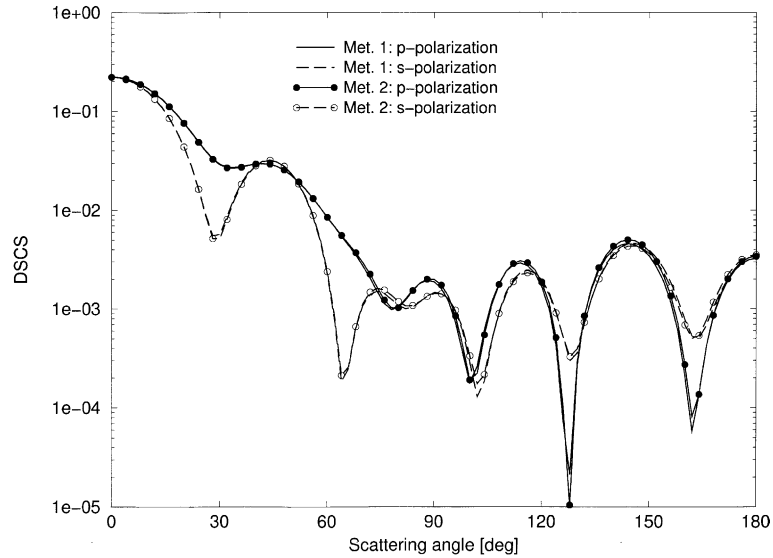


Fig. 1. Differential scattering cross-sections for a prolate scatterer with $k_s a = 8$ and $k_s b = 4$. The results are computed with the null-field method with circularly distributed spherical vector wave functions (Met. 1) and localized spherical vector wave function (Met. 2). For the method with distributed sources we used $k_s r_0 = 1$, $M = 2$ and $N = 6$, while for the method with localized sources we used $N = 13$.

combination of circularly distributed and localized spherical vector wave functions are shown in Fig. 3. For the first method, the sources were

distributed in the xy -plane, on a circle of radius $k_s r_0 = 4$, while for the second method the sources were distributed on a circle of radius $k_s r_0 = 6$. Also

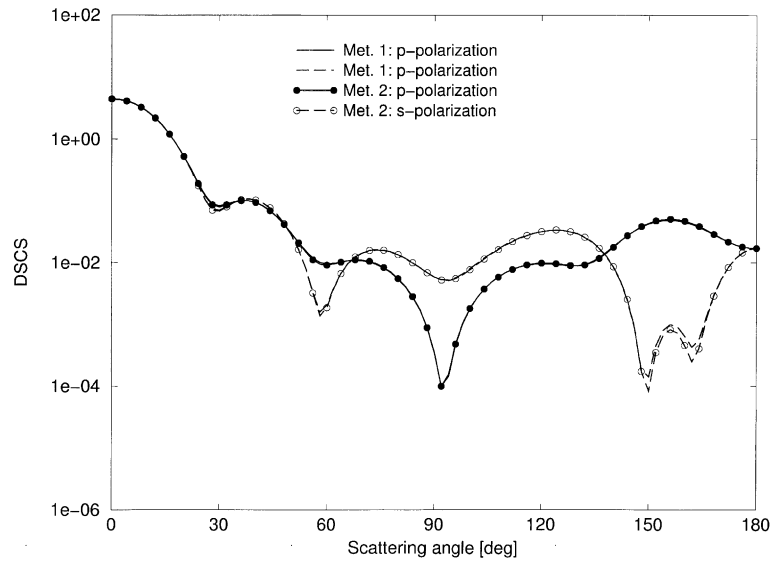


Fig. 2. Differential scattering cross-sections for an oblate scatterer with $k_s a = 4$ and $k_s b = 8$. The results are computed with the null-field method with circularly distributed spherical vector wave functions (Met. 1) and localized spherical vector wave functions (Met. 2). For the method with distributed sources we used $k_s r_0 = 4$, $M = 2$ and $N = 6$, while for the method with localized sources we used $N = 13$.

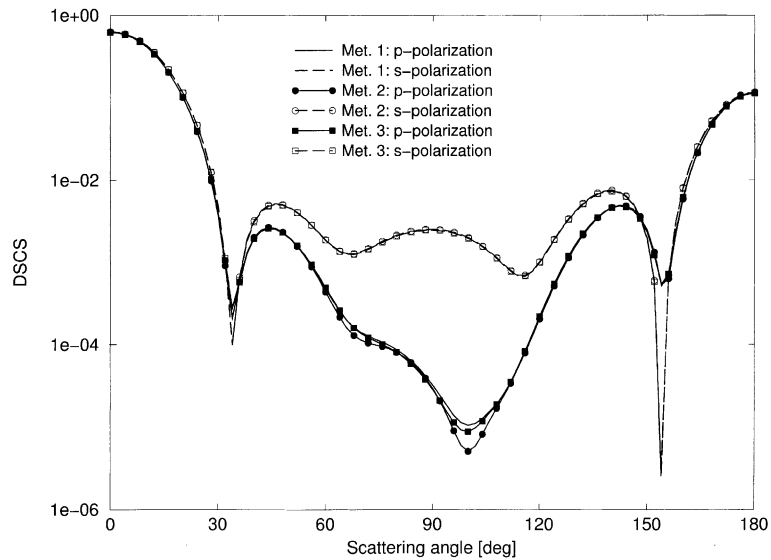


Fig. 3. Differential scattering cross-sections for an oblate scatterer with $k_s a = 1$ and $k_s b = 8$. The results are computed with the null-field method with circularly distributed spherical vector wave functions (Met. 1), a combination of circularly distributed and localized spherical vector wave functions (Met. 2) and spherical vector wave functions distributed in the complex plane (Met. 3). In the first case, we used $k_s r_0 = 4$, $M = 2$ and $N = 6$, in the second case we used $N = 6$ for the localized source and $k_s r_0 = 6$, $M = 2$ and $N = 3$ for the circularly distributed sources, while in the third case we used 13 sources distributed in the complex plane.

shown are the results corresponding to the null-field method with spherical vector wave functions distributed in the complex plane. Note, that in this case the null-field method with localized spherical vector wave functions fails to converge. The agreement between the scattering characteristics is acceptable.

4. Conclusions

A novel formulation of the null-field method with circularly distributed spherical vector wave functions has been proposed. This system of functions is attractive for computing the light scattered by flattened axisymmetric particles due to the $\exp(jl\varphi)$ - and $|\mathbf{r} - \mathbf{r}_0|$ -dependence of the vector functions. As a result, the amount of

computer time and storage are significantly reduced and a stable approximation of the solution is obtained. Combining this system with the system of localized spherical vector wave functions, particles with complex structures can be computed.

References

- [1] P.C. Waterman, Phys. Rev. D 3 (1971) 825.
- [2] A. Doicu, Y. Eremin, T. Wriedt, Acoustic and Electromagnetic Scattering Analysis Using Discrete Source, Academic Press, London, 2000.
- [3] P. Leuchtman, Linienmultipole: Herleitung der Formeln. Internal report of the Institut für Feldtheorie und Hochfrequenztechnik, ETH, 1994.
- [4] C. Hafner, L. Bomholt, Electrodynamical Wave Simulator, John Wiley, Chichester, 1993.