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Scattering of evanescent waves by a sensor tip near a plane surface

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Abstract

The paper is devoted to the application of a rigorous approach for analyzing the evanescent wave scattering by sensor tip near a plane surface. On the basis of the discrete sources method a numerical schemes was developed and implemented in computer program. Numerical results related to the influence of the position of the sensor tip on the scattering characteristics are discussed. © 2001 Published by Elsevier Science B.V.

Keywords: Evanescent wave scattering; Sensor tip scattering; Photon scanning tunneling microscope; Discrete sources method

1. Introduction

The recent development in scanning near-field optical microscopy (SNOM) [1] has extended the spatial resolution to subwavelength regions [2]. This gave rise to the photon scanning tunneling microscopy (PSTM). This technique requires a sensor tip, which is able to convert evanescent waves into propagating waves. The resulting scattered field can be detected in the far zone by conventional optical detectors, while a computer image of the relief is reconstructed by a raster scanning analysis. It has been established that a dielectric tip or even an atomic force microscopy (AFM) sensor are able to convert evanescent waves into scattered waves. In this context it is it possible to control the position of the sensor tip in the vicinity of the examined surface for obtaining a spatial resolution far beyond the diffraction limit of the conventional optical microscope [3]. The wide potential application of this technique in biology, medicine, material science and technology requires the scattering analysis of evanescent waves by a penetrable scatterer (sensor tip) near the plane surface of a dielectric prism. The most 3-D models do not completely account for the interaction between the local scatterer and the plane interface [4,5]. Since recently computer simulations [6] demonstrate that this interaction leads to significant changes in the scattering characteristics, we will use in this paper a rigorous model for evanescent wave scattering based on the discrete sources method (DSM) [7]. We will investigate the conversion of evanescent wave into scattered

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waves for a sensor tip similar to that used by Girard and Dereux [8]. By computer simulation we will analyze the possibility to control the position of the sensor tip above the plane dielectric surface using conventional optics.

2. Discrete sources method

The geometry of the scattering problem is shown in Fig. 1. An axisymmetric particle with a smooth boundary S and interior D_i is situated on a plane surface Σ, so that its symmetry axis coincides with the normal to the plane surface. The upper half-space corresponding to the ambient medium is denoted by D_0, while the lower half-space corresponding to the glass prism is denoted by D_1. Let us introduce a rectangular coordinate system Oxyz by choosing the origin O at the tangent point between the particle and the substrate. The z-axis coincides with the symmetry axis of the particle and is directed into the domain D_0. The wave number in the domain D_t, $t = 0, 1, i$, is denoted by $k_t = k(\epsilon_t \mu_t)^{1/2}$. Similarly, n_t, $t = 0, 1, i$, stands for the index of refraction of the domain D_t. The external excitation $\mathbf{E}_{\text{inc}}$, $\mathbf{H}_{\text{inc}}$ is a polarized plane wave propagating in the glass prism at the angle β_1 with respect to the z-axis. The refracted wave is propagating in D_0 at the angle β_0, according to the Snell's law. The mathematical formulation of the scattering problem consists in the Maxwell equations

$$\begin{aligned} \nabla \times \mathbf{E}_t &= jk\mu_t \mathbf{H}_t, \\ \nabla \times \mathbf{H}_t &= -jk\epsilon_t \mathbf{E}_t, \quad \text{in } D_t, \ \ t = 0, 1, i, \end{aligned} \tag{1}$$

the transmission conditions at the plane interface and the particle surface

$$\begin{aligned} \mathbf{e}_z \times (\mathbf{E}_1 - \mathbf{E}_0) &= 0, \\ \mathbf{e}_z \times (\mathbf{H}_1 - \mathbf{H}_0) &= 0 \ \text{ on } \Sigma, \end{aligned} \tag{2}$$

and

$$\begin{aligned} \mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_0) &= 0, \\ \mathbf{n} \times (\mathbf{H}_i - \mathbf{H}_0) &= 0 \ \text{ on } S, \end{aligned} \tag{3}$$

respectively, and the radiation (attenuation) condition at infinity. Here, $\mathbf{n}$ is the outward unit normal vector to S and $\mathbf{E}_t$, $\mathbf{H}_t$ stands for the total field in the domain D_t. Note that the total field in D_0 sums the contribution of the refracted incident field and the scattered field, that is $\mathbf{E}_0 = \mathbf{E}_{\text{sca}} + \mathbf{E}_{\text{inc}}^{\text{ref}}$ and $\mathbf{H}_0 = \mathbf{H}_{\text{sca}} + \mathbf{H}_{\text{inc}}^{\text{ref}}$.

We will construct an approximate solution to the scattering problem by representing the electromagnetic fields as a finite linear combination of fields of multipoles [9]. The approximate solutions satisfy the Maxwell equations in the domains D_t, $t = 0, 1, i$, the radiation condition in the domains D_t, $t = 0, 1$, and the transmission condition at the plane interface. Essentially, the scattering problem simplifies to the approximation problem of the external excitation on the particle surface. For an axisymmetric sensor tip it is possible to construct an approximate solution using the technique described in Refs. [10]. This technique can be summarized as follows:

1. The scattered fields representation is based on the Green's tensor for the half-space. Consequently, the vector potentials of the multipole fields are expressed as Weyl–Sommerfield integrals [9,10]. This representation provides the continuity of the tangential component of the scattered fields at the plane interface.
2. For the internal field representation we used vector potentials of regular multipole fields.
3. The multipoles are distributed along the axis of symmetry inside D_i or in the image domain of

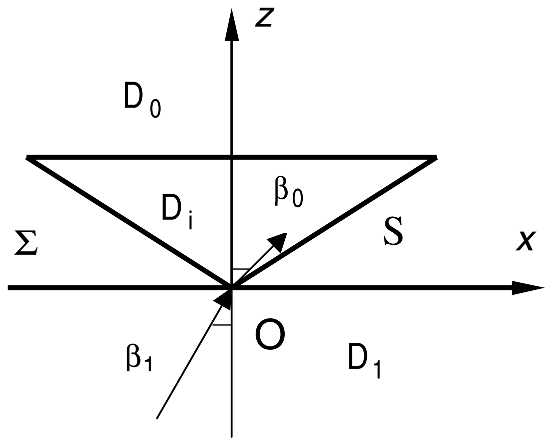

Fig. 1. Geometry of the scattering system.

the complex plane [9]. Consequently, the approximate solution is represented as a linear combination of Fourier harmonics with respect to the azimuthal angle. The approximation problem of the external excitation simplifies to a sequence of one-dimensional approximation problems along the scatterer generator. This approach significantly reduces the computational effort.
4. The completeness of the system of distributed multipoles guarantee the convergence of the approximate solution to the exact solution in closed subsets of D_0 [9].

As mentioned before the approximate solution fulfilled the conditions of the scattering problem except the transmission condition at the particle surface. In fact this condition will be used to determine the amplitudes of discrete sources. For this purpose it is necessary to compute the refracted plane wave in D_0. In the case of a p-polarized plane wave propagating at an angle β_1 with respect to Oz-axis (the electric field vector belongs to the incident plane Oxz) we have

$$\mathbf{E}_{\mathrm{inc}}^{\mathrm{ref}} = t_{\mathrm{p}}(-\mathbf{e}_x \cos\beta_0 + \mathbf{e}_z \sin\beta_0) \times \exp[-\mathrm{j}k_0(x\sin\beta_0 + z\cos\beta_0)],$$

$$\mathbf{H}_{\mathrm{inc}}^{\mathrm{ref}} = -t_{\mathrm{p}}\mathbf{e}_y \exp[-\mathrm{j}k_0(x\sin\beta_0 + z\cos\beta_0)].$$

Here ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) are the unit vectors of the Cartesian coordinate system and t_{p} is the transmission Fresnel coefficient for p-polarization

$$t_{\mathrm{p}} = \frac{2n_1\cos\beta_1}{n_0\cos\beta_1 + n_1\cos\beta_0}.$$

If $\beta_1 > \arcsin(n_0/n_1)$ the incident wave will be totally reflected and only a damped or an evanescent wave traveling along the surface will be present in the upper half-space. In this case it follows from Snell's law that $\sin(\beta_0) > 1$, hence $\cos(\beta_0)$ becomes purely imaginary. We choose $\cos(\beta_0) = -\mathrm{j}(\sin^2 \times (\beta_0) - 1)^{1/2}$, since otherwise the amplitude of the refracted wave would tend to infinity with increasing distance.

In the case of a s-polarized plane wave we have

$$\mathbf{E}_{\mathrm{inc}}^{\mathrm{ref}} = t_{\mathrm{s}}\mathbf{e}_y \exp[-\mathrm{j}k_0(x\sin\beta_0 + z\cos\beta_0)],$$

$$\mathbf{H}_{\mathrm{inc}}^{\mathrm{ref}} = t_{\mathrm{s}}(-\mathbf{e}_x \cos\beta_0 + \mathbf{e}_z \sin\beta_0) \times \exp[-\mathrm{j}k_0(x\sin\beta_0 + z\cos\beta_0)],$$

where the transmission Fresnel coefficient for s-polarization is given by

$$t_{\mathrm{s}} = \frac{2n_1\cos\beta_1}{n_1\cos\beta_1 + n_0\cos\beta_0}.$$

Various schemes for amplitude determination are at our disposal. As it has been found stable results can be obtained by using the generalized point-matching technique and the pseudoinversion of an over-determined system of linear equations. The DSM is a direct method and therefore it allows to solve the scattering problem for all set of incident angles β_1 and for all p- and s-polarizations at once. Besides, the numerical scheme provides an opportunity to control the convergence of the approximate solution by a posterior error estimate [9]. After the amplitudes of discrete sources have been determined one can calculate the far-field pattern $\mathbf{E}_{\mathrm{s}\infty}$ of the scattered field $\mathbf{E}_{\mathrm{s}}$

$$\mathbf{E}_{\mathrm{s}}(\mathbf{r}) = \frac{\exp(-\mathrm{j}k_0 r)}{r}\mathbf{E}_{\mathrm{s}\infty}(\theta,\varphi) + \mathrm{O}\left(\frac{1}{r^2}\right),$$

$$\text{as } r \to \infty \text{ and } z > 0,$$

by using the asymptotic representation for the Sommerfield integrals [9].

3. Results and discussion

We consider a polarized plane wave with a wavelength of $\lambda = 0.488$ mm in free space. We assume that the scatterer is deposited near the glass substrate and that the refractive index of the substrate is $n_1 = 1.5$. In this case the evanescent waves appear for incident angles $\beta_1 > 41.81$. We will analyze the differential scattering cross section for p- and s-polarization

$$\sigma_{\mathrm{d}}^{\mathrm{p,s}}(\beta_1,\theta,\varphi) = \left|E_{\theta\infty}^{\mathrm{p,s}}(\beta_1,\theta,\varphi)\right|^2 + \left|E_{\varphi\infty}^{\mathrm{p,s}}(\beta_1,\theta,\varphi)\right|^2,$$

and the integral response

$$R^{\mathrm{p,s}}(\beta_1) = \int_{\Omega} \sigma_{\mathrm{d}}^{\mathrm{p,s}}(\beta_1,\theta,\varphi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi.$$

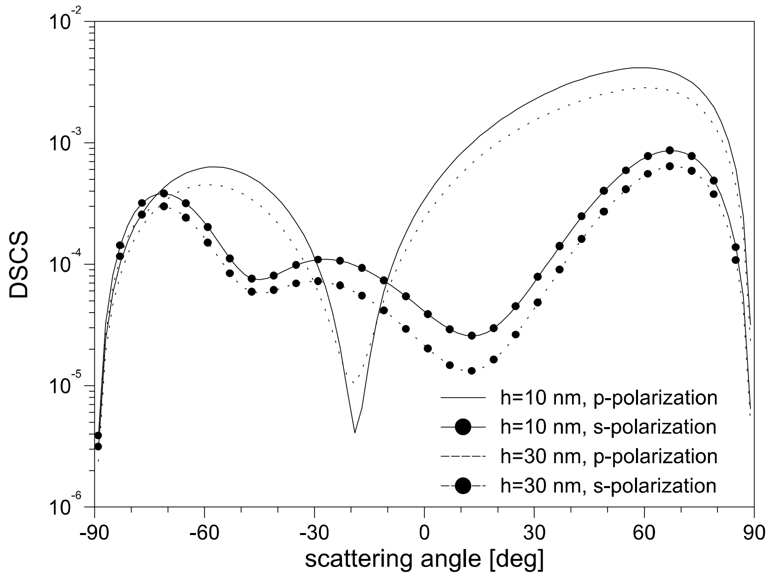

Fig. 2. Differential scattering cross section for p- and s-polarization for a Si sensor placed at two different distances $h = 10$ nm and $h = 30$ nm above the substrate. The incident angle is $\beta_1 = 60°$.

Essentially, the integral response represents the intensity of the scattered light collected by an optical system in the solid angle Ω.

In our simulations we consider the sensor tip to be a cone with a diameter of $D = 0.20$ mm, a tip angle of $\alpha = 30°$ and a tip radius of $r = 30$ nm.

In Fig. 2 we show the differential scattering cross section for a Si sensor ($n_i = 4.37 - 0.008j$) placed at two different distances $h = 10$ nm and $h = 30$ nm above the substrate. The differential scattering cross section is computed in the azimuthal planes $\varphi = 0°$ and $\varphi = 180°$ for $0 \leqslant \theta \leqslant 90°$. By convention, the scattering angle θ in the azimuthal plane $\varphi = 180°$ will be denoted by $-\theta$. Since the intensities for p-polarization exceed the intensities for s-polarization almost over the entire scattering domain we will concentrate only on the case of p-polarization. The differential scattering cross sections for p-polarization and different positions of the sensor tip are depicted in Fig. 3. The results show that the position of the sensor can be controlled by optical systems centered at the scattering directions $\theta = \pm 50°$.

Let us consider two spherical lenses having their centers located at the scattering directions $\theta_1 = 50°$, $\varphi_1 = 180°$ and $\theta_1 = 50°$, $\varphi_1 = 0°$. The aperture angle of both lenses is $\theta_a = 20°$. In Fig. 4 we plot the integral response for different positions of the sensor tip. The curves are almost linear and show a decrease of the integral response when the distance between the sensor and the interface increases.

Simulation results for a SiN sensor ($n_i = 2.0$) are shown in Figs. 5–7. As before, it is clear that the position of the sensor tip can be controlled by optical systems centered at the scattering directions $\theta = \pm 50°$.

Fig. 8 shows the p- and s-polarized integral response in the upper hemisphere ($0 \leqslant \theta \leqslant 90°$, $0 \leqslant \varphi \leqslant 360°$) versus the incident angle. The p-polarized response exceeds the s-polarized response almost over the entire domain of the incident angle. To maximum value of the integral response is achieved for $\beta_1 = 50°$. This value of the integral response is several times bigger than the value corresponding to $\beta_1 = 60°$, which is used by the majority of researchers [4–6].

4. Conclusion

A rigorous model based on the DSM has been used to investigate the evanescent wave scattering

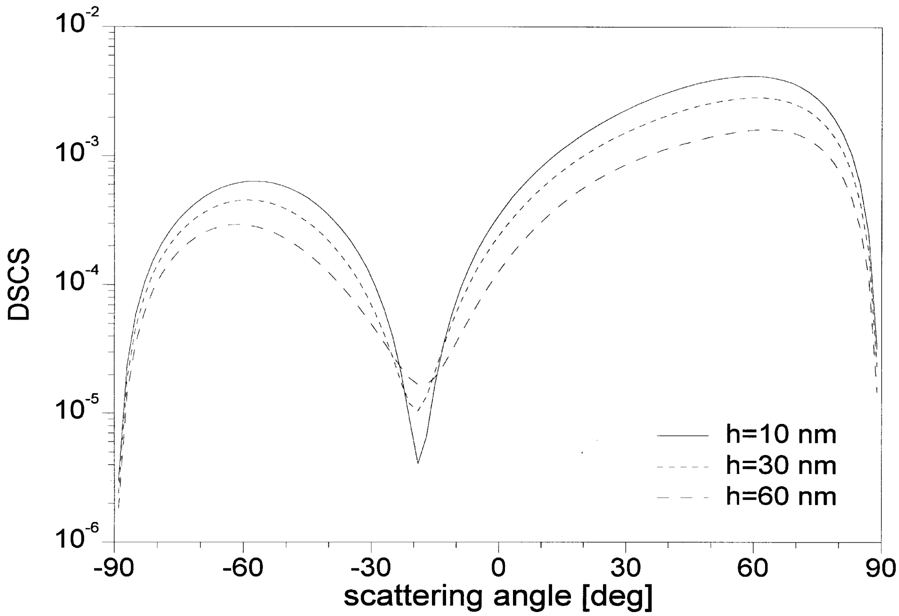

Fig. 3. Differential scattering cross section for p-polarization for a silicon sensor placed at three different distances $h = 10$ nm, $h = 30$ nm and $h = 60$ nm above the substrate. The incident angle is $\beta_1 = 60°$.

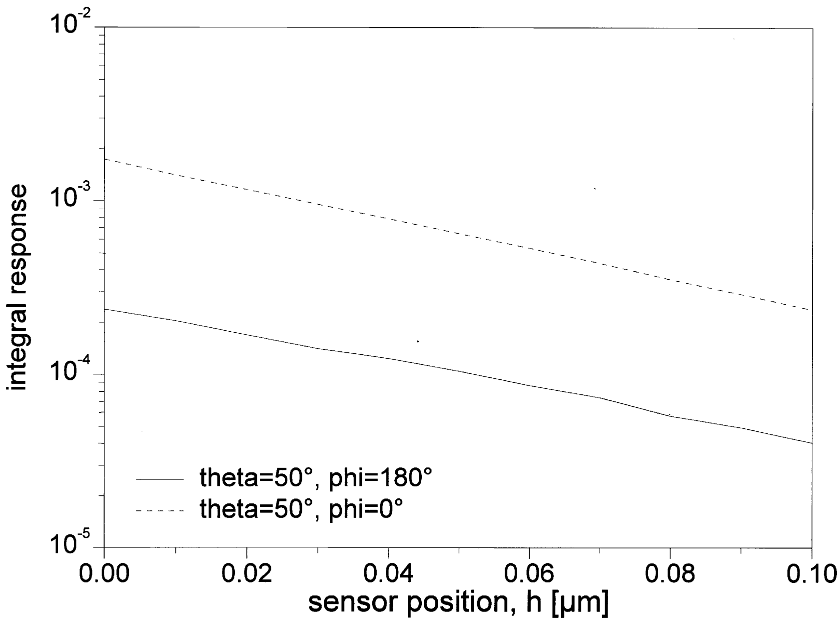

Fig. 4. Integral response of two optical system placed at $\theta_1 = 50°$, $\varphi_1 = 180°$ and $\theta_1 = 50°$, $\varphi_1 = 0°$ for a Si sensor. The distance between the sensor and the substrate varies between 0 and 100 nm. The aperture angle of the optical system is $\theta_a = 20°$ and the incident angle is $\beta_1 = 60°$.

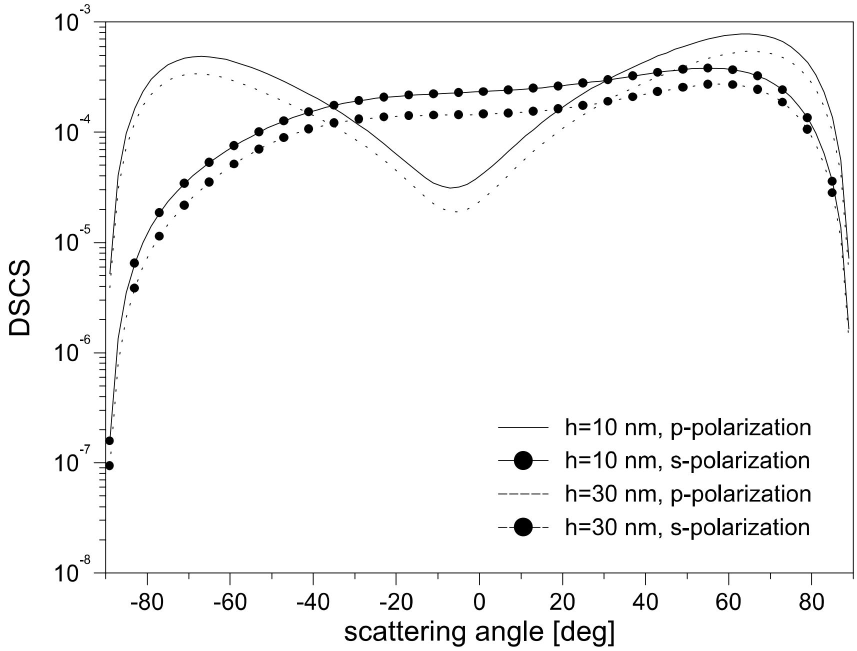

Fig. 5. The same as in Fig. 2 but for a SiN sensor.

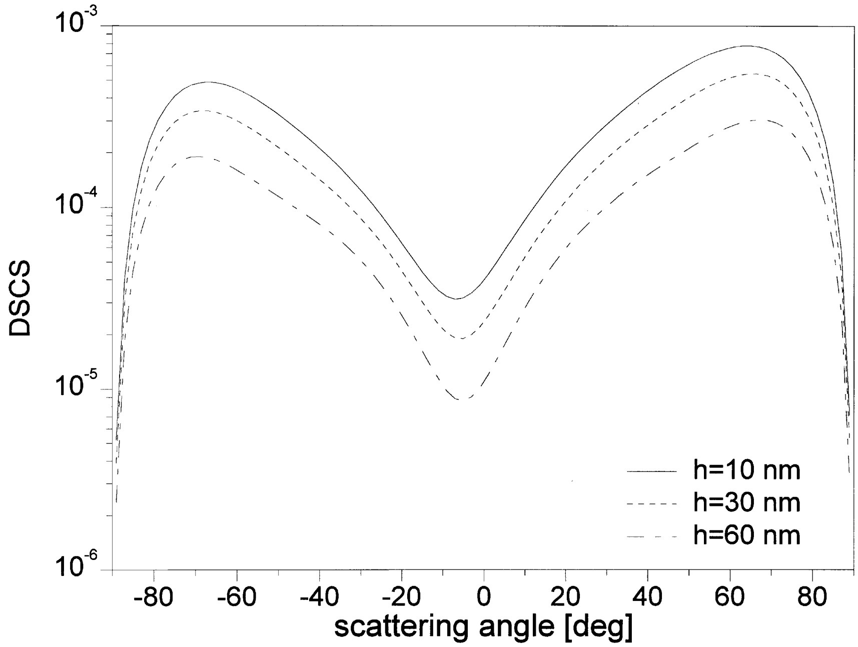

Fig. 6. The same as in Fig. 3 but for a SiN sensor.

from a sensor tip near a plane interface. The analysis of the differential scattering cross section and of the integral response shows that the position of the sensor tip can be efficiently controlled.

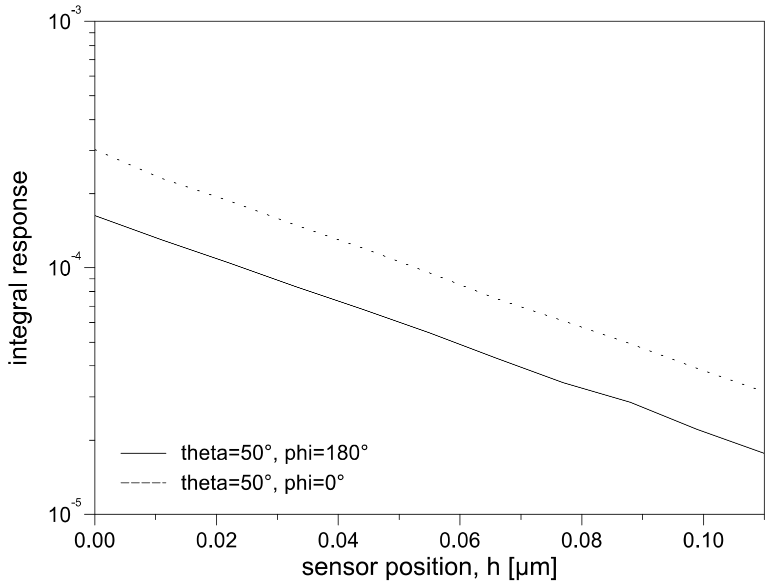

Fig. 7. The same as in Fig. 4 but for a SiN sensor.

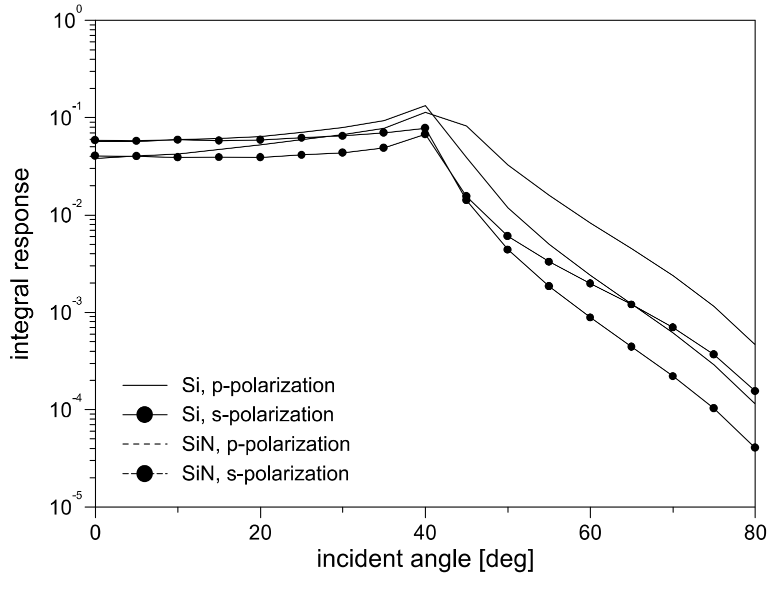

Fig. 8. Total integral response in the upper hemisphere as a function of the incident angle.

In addition, our simulation demonstrate that the integral response of an optical system can be increased by an appropriate choice of the incident angle.

Acknowledgements

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